DAMAGE BOOK

STAR CHARTS.

THOMASON
CIVIL ENGINEERING COLLEGE
MANUALS,

(COMPILED BY THE COLLEGE STAFF).

No. XIV.

SURVEYING.

(PART II.)

C. J. VEALE, F.R.A.S., F.R.G.S.,

Professor of Surveying and Drawing.

TWELFTH EDITION.

ROORKEE:

PHOTO.-MECH. AND LITHO. DEPARTMENT, THOMASON COLLEGE.

1926.

(On sale at the BOOK DEPOT, Thomason College, Roorkee).

PREFACE.

In the second edition of this volume the chapter on Military Surveys and reconnaissance has been omitted in favour of a chapter on Hydro-Electric Surveys. Other chapters have been brought up to date and revised where necessary.

ROORKEE : October 1926. } C. J. VEALE, F.R.A.S., F.R.G.S.
Professor of Surveying and Drawing.

CHAPTER I.

TRIGONOMETRICAL SURVEYING OR TRIANGULATION.

1. **Triangulation.***—The basis of an accurate survey must necessarily be an *extended system of Triangulation*, the preliminary step in which is the careful measurement of a base line on some level plain. At each extremity of this base, the angles are observed between several surrounding objects previously fixed upon as Trigonometrical Stations, and also those subtended at each of these points by the base itself. The distances of these stations from the end of the base line and from each other are then calculated, and laid down upon paper, forming so many fresh bases from whence other trigonometrical points are determined, until the entire tract of country to be surveyed is covered over with a net-work of triangles, of as large a size as is proportional to the contemplated extent of the survey, and the quality and power of the instruments employed. The interior detail between these points is filled up either by measurement with the chain and theodolite, chain and prismatic compass, or by the plane-table method given in Chapter VI., Part I.

For the description of a regular Trigonometrical Survey of a country the reader must refer to larger works. What will be described here is such a survey as might be made with a 5-inch theodolite, if the surveyor had some few square miles of country to survey accurately.

The following general description of the process will be clearer by reference to *Plate* I. The calculations, it will be seen as the explanation goes on, though simple enough, are somewhat confusing, and it is necessary they should be made regularly in proper form,† both to ensure accuracy in the first instance and to allow of their being checked afterwards by another calculator.

2. **Base Line.**—In fixing upon an appropriate *site* for the measurement of a base line, a level piece of ground should obviously be selected, where both ends of the base would be visible from the nearest trigonometrical points. It should also be as near the centre of the survey as possible, but this is not absolutely necessary. For a survey

* Chapter III., Part I of this Manual should be consulted concerning the Theodolite.

† With the Surveyor-General's permission Survey of India Forms have been used where they suit the same.

No. 420—1500.

of the extent above mentioned, it should be about 2,000 feet long, and the sides of the triangles may be increased to a mile or more. Thus, in the skeleton plot, *see* Plate I., AB was selected as the base line.

Measuring the Base line.—This operation, being the basis of the survey, must be performed with as great exactness as the instruments available will allow. For this reason the slope of the ground should be measured in order to reduce the surface measurement to its horizontal equivalent, and if there are changes of slope, the points at which these changes occur should be noted and the different slopes recorded as shown in Form A. These slopes are measured by setting up the theodolite at one end of the slope, and sending to the other end a staff with a vane set to the height of the instrument. Any error in zero of altitude in the instrument should be eliminated, either by measuring the inclination in both directions and taking the mean reading, or by observing the inclination twice from one point once with the face reversed, and taking the mean of the readings. With a number of changes of slope a saving of time and trouble will be effected by adopting the latter method and setting up the theodolite at alternate points only.

In measuring a base with an ordinary chain, as in this class of survey, the chain should be compared with a standard and made correct in length before measurement. It should be again tested after measurement, and if found to have varied, the measurement must be rejected.

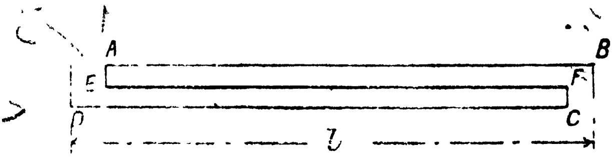
The base should be measured and re-measured the same day, and the mean of the two lengths taken. If too long to admit of this being done it should be divided into suitable sections.

The variation in length, if any, between the two measurements should be shown only as occurring in the last slope, and the measurements should be continuous throughout the base or the section done in one day, as the case may be. This will possibly involve a slight variation in the position of the points at which the slope is taken to change, but this variation will be so slight as to produce no appreciable error in the deduced horizontal length. Form A, the method of recording the measurement of the base when taken with an ordinary chain, has been changed in this edition. The form here given is one which has been in use of late years by students in making a small trigonometrical survey in Roorkee. Columns for the record of actual measurements only are given to avoid the occurrence of calculations in the Field-book. In the column of remarks the tests applied to the chain should be stated with their result. The method of measuring the excess or defect of the base length from an integral number of chain lengths should also be stated. For very accurate work and for extended

operations, the base line measurement should be reduced to a value at M. S. L. (*see* Appendix VIII.)

Some of the most accurate base lines have been measured by Bessel's compensated rods, and a description of the rod would not be out of place here.

Fig. 1.



AB and CD are *steel* rods (*Fig.* 1) each nearly measuring the required length l. They are fixed by the pieces AE and CF to the ends E and F respectively of a *zinc* rod EF. The expansion of AB and CD, tending to increase l is counteracted by the expansion of EF, which tends to decrease the spaces AD and BC, that is, to pull the ends B and D in towards the centre of the rod. In the compensated rod the zinc exactly counters the steel. This may be effected thus—

Expansion of steel $= \frac{1}{840}$ of its length for 180°.

Expansion of zinc $= \frac{1}{344}$ of its length for 180°.

Then total expansion of steel $= \frac{AB + CD}{840}$ in 180°,

and total expansion of zinc $= \frac{EF}{344}$ in 180°.

But $l + EF = AB + CD \therefore \frac{l + EF}{840} = \frac{EF}{344}$

whence EF $= \frac{344\,l}{496}$, so that if $l = 10$ feet, EF or length of zinc rod $=$

$\frac{3440'}{496} = 6$ feet 11·2 inches.

For ordinary base line work mean results with a steel tape should suffice. Greater accuracy than this can be had by using an INVAR tape which gives an accuracy of $\frac{1}{10}$-inch per mile, and INVAR rods an accuracy as low as $\frac{1}{2,500,000}$ of the measured length or about $\frac{1}{40}$th inch to the mile.

Form A. (FIELD-BOOK).

MEASUREMENT of Base line on Maidan, Roorkee, with 100 feet chain.

Date of measurement ________________

From	To	Distance.	Vertical angle. Degrees.	A. ′	A. ″	B. ′	B. ″	Remarks.	
A	1	300	+ 0	9	30	10	0	*Rising ...	*Chain before measurement 100·00 feet; after measurement 100·10 feet; mean 100·05 feet.
		300	+ 0	10	0	9	0		
1	2	600	+ 0	3	0	2	30	Rising ...	
		600	+ 0	3	0	3	0		
2	3	300	− 0	47	0	47	0	Falling ..	
		300	− 0	46	30	47	0		
3	4	400	+ 0	26	0	27	0	Rising ...	Excess over 19 chains measured with levelling staves
		400	+ 0	26	0	26	0		
4	B	318·56	− 0	19	30	19	30	Falling ...	
		318·75	− 0	19	0	19	0		

N. B.—The sign + or − is given according as the vertical angle is one of elevation or depression *in the direction of the measurement*

*As before stated the chain should be correct before and after measurement This difference is, however, shown here to avoid changing all the calculations of former editions. It also serves to illustrate the method of applying the necessary correction.

Form B. (CALCULATION BOOK).

Reduction of Base Line.

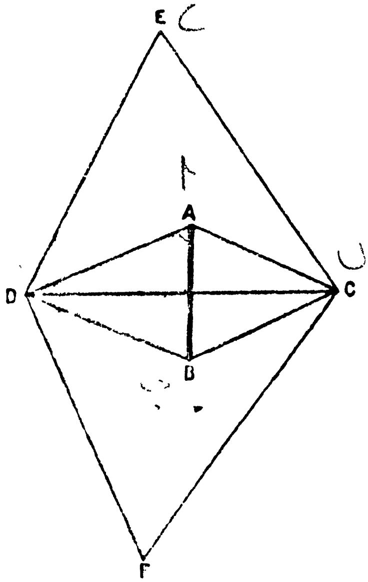
Stations.	Measured distances in feet	Inclination of ground.	Logarithmic computations.		Horizontal distances in feet	Relative altitudes in feet.	Reduced levels.	Remarks.
		° ′ ″						
1		+0 9 30	Log. cos.	$\bar{1}$·9999983			100·000	North end of base.
	300			2·4771213				
				2·4771196	299·999			
			Log. tan.	$\bar{3}$·4467518				
				$\bar{1}$·9238714		+ ·839	100·839	
2		+0 2 52		$\bar{1}$ 9999998				
	600			2·7781513				
				2·7781511	600·000			
				$\bar{4}$·9211034				
				$\bar{1}$·6992545		+ ·500	101·339	
3		−0 46 52		$\bar{1}$ 9999596				
	300			2 4771213				
				2·4770809	299·972			
				$\bar{2}$·1346171				
				0·6116980		−4·090	97·249	
4		+0 26 15		$\bar{1}$·9999873				
	400			2 6020600				
				2 6020473	399·988			
				$\bar{3}$·8828639				
				0·4849112		+3·054	100·303	
5		−0 19 15		$\bar{1}$·9999932				
	318·655			2·5033208				
				2·5033140	318·650			
				$\bar{3}$·7481614				
				0 2514754		−1·784	98·519	South end of base.
					1918·609			

Length of chain = 100·05 feet ∴ true length of Base Line = $\frac{1918·609 \times 100·05}{100·00}$ = 1919·57 feet.

3. **Well-conditioned Triangles.**—The trigonometrical stations must be chosen with a view to the formation of *well-conditioned* triangles, *i.e.*, triangles none of whose angles is less than 30°; the nearer the triangle approaches to the equilateral the better. The sides of the triangles should increase as rapidly as possible from the measured base. The accompanying sketch (*Fig.* 2) shows how this is to be managed without admitting any *ill-conditioned* triangles.

AB is supposed to be the measured base, and C and D the nearest trigonometrical points. All the angles being observed and the length of AB having been measured, the other sides of the triangles DAB, CAB may be calculated. DC can then be obtained from the two triangles DAC, DBC (having the two sides and included angle of each given), one calculation acting as a check upon the accuracy of the other. This line DC is again made the base from which the distances of the trigonometrical stations E and F are computed from D and C, and these lines ED, EC, DF, CF, can be used as fresh bases for extending the triangulation, or if these be not sufficiently large, the length of EF can be calculated and used as a base.

Fig. 2.



This is the usual method of starting from a base line, unless the nature of the ground to be surveyed interferes.

4. **Stations.**—The remainder of the trigonometrical stations must be arranged over the whole survey, as the nature of the country will best allow, and care must be taken that no point in the survey is too far from some one of these stations.

5. **Signals.**—Signals are of two kinds—luminous and opaque. Luminous signals are either heliotropes or lamps, and the reflected light of the sun or the light of the lamp is directed through a sight vane which is plumbed over the station mark.

The heliotrope is a circular mirror with the silver backing scraped off at the centre spot which is the peep-hole for aligning. When the sight vane is aligned through this peep-hole and the mirror is slanted towards

the sun so that the sun's rays are reflected on to the vane, the unsilvered portion shows as a black dot, and when this dot is in the line of sight the sun's reflection is carried to the observer.

The best opaque signal is a pole and brush, or grass bound in the shape of a brush or sometimes a cross. A pair of common wicker baskets, placed mouth to mouth, with the pole passed through, makes an excellent signal.

The pole is plumbed vertically over the station mark and a cairn of stones built up to steady it. The cairn might, with advantage, be whitewashed if it is over a station in the jungle or low ground as it will then show up against a dark background.

It sometimes happens in hilly country that a station in the plain is very valuable for the reintersection of certain auxiliary points, when a bell-shaped "shuldari" or followers'-tent makes an excellent mark.

Trees which have been brushed or flagged make good signals.

6. **Observing angles.***—Having selected all the stations and placed the signals, all the angles of the triangles must be observed with the theodolite; and to obtain the relative altitudes of the ground at the different stations, the vertical angles must also be read. The angles are observed from each station in succession, as follows:—The theodolite is centered over the point on the ground, marking what is called the station dot, and which is vertically under the signal. When tha signal is high, this is found in the following manner:—Set up the theodolite at a short distance from the signal, and having levelled it, fix the intersection of the wires on the signal, clamp the lower plates, and bring the telescope down till it intersects the ground a foot or so beyond the signal. Make a mark at this point, and stretch the chain from the theodolite to it. Now remove the theodolite to another point, so that the direction between it and the signal is about at a right angle with the last direction, repeat the operation; the point where these two lines intersect will be vertically under the signal.

Place the legs of the theodolite stand firmly into the ground and examine it for any shake. Next plumb the instrument over the station dot and level up with footscrews. If the work is to be computed or based on a magnetic bearing attach the magnetic compass; set the plates to read 0°, unclamp C_1 (*see* Plate 2) and rotate instrument till the compass needle points N. and S. Clamp C_1 and unclamp C_2 and intersect and read the zero station. This reading will be the direction of the zero station from magnetic north, or in other words, it will be the magnetic bearing of the zero station, which enter in Form C.

*On geodetic operations the measurement of angles with modern instruments has been reduced to ± ½ second.

(*a*). **The zero station** is the station selected by the observer as the station from which he commences and on which he ends his rounds of observations.

(*b*). **Setting to zero** is a technical expression meaning that the zero station has been set to some particular reading. When a theodolite has only two verniers, a change of face does not mean a change of zero, as the verniers are opposite and become interchanged, but with a three-vernier instrument each change of face means a change of zero.

(*c*). The rule about zeros is as follows:—If the zero station was originally set to 0° then the next zero will be set to $\frac{360^\circ}{\text{No of verniers} \times \text{No of sets of angles}}$

This changing of zeros or observing on one, two, three or more zeros tends to eliminate any errors that exist in the graduation of the divisions of arc, as the angles are read on different portions of it. For the purpose of this work and for engineering practice generally, 0° and 90° may be taken as the settings for two zeros and two zeros will be found sufficient.

7. **Form C**.—This form is a copy of the triangulation field-book, and one page allows space for observation of horizontal angles on two zeros 0° and 90°, and two faces for each zero, also vertical angles on two faces.

Procedure.—Clamp the upper plate C_2 so that A vernier is on "face" right (that is, the vernier below the arc) and is set to 0° 0′ 0″, or better still to some small amount slightly greater than 0° 0′ 0″; and by unclamping the lower plate direct the telescope to intersect the zero station. Intersect by slow-motion screw T_1 of lower plate after clamping it by the clamp C_1. The lower plate will not now be touched till two faces have been read on this zero. Unclamp upper plate and read to the next station by approaching it gradually without overshooting it in the direction of the hands of a clock, known as right "swing," read and register and so on, finally coming back again on to the zero station. Care must be again taken to approach the zero station gradually and not to pass it, but with the slow-motion screw T_2 to intersect it. Read and register. Now unclamp *upper* plate, transit telescope, and bring it back to the zero station without overshooting it, in a direction *contrary* to the hands of a clock known as left "swing," and continue in this direction reading the horizontal angles with the same caution of not overshooting the signals, finally closing again on the zero station and registering the angles for L 180 from the *bottom* line of the column upwards. This constitutes one set. The upper clamp is now loosened and the telescope rotated vertically and A vernier is set to 90° 0′ 0″, or something greater, the upper C_2 clamp is now fastened, the lower clamp

C_1 loosened, and the zero station brought into the field of view in a direction of the hands of a clock, and observations treated in the same way as with the other set.

The means are taken out by considering the values of the zero, station 0° 0′ 0″, and subtracting the mean of the starting and closing value as registered from the other values of that observation The "General Mean" is a mean of these means.

The observations are taken on both "faces" to eliminate horizontality of telescope axis error—on two or more zeros to eliminate graduation error, and on a right and left "swing" or in contrary directions to eliminate the error in "drag" of the verniers on the limb.

Vertical angles should *not* be read at the same time as the horizontal readings. The procedure of reading and recording these angles is a straightforward one—A vernier being invariably the vernier towards the object end of the telescope.

Having completed the vertical angles enter the height of the axis of the telescope and the height of the signal from the station mark, and write a clear and concise description of the station so that there will be no doubt as to the position of the mark which is very often buried out of sight to avoid destruction.

Cautions to be observed in triangulation.

Set up instrument *exactly* over station dot, especially in the presence of followers, khalasis, etc., who, if they find you are casual and careless about this, will never trouble to give a carefully plumbed mark or a helio in true position. Unhook plummet, as if left in a wind, it will set up a vibration. Level up theodolite, lightly brush the verniers with a soft brush, and only give enough pressure to the clamp screws to make them grip. A clamp screw should be tightened only sufficiently so that a gentle pressure in the reverse direction will ease it without jerk or jump on the instrument to the great detriment of angular work; in fact good angular work is more a matter of hands than eye, and for this reason turn the instrument gently by handling the upper and lower plates—*not the telescope.* In fact cultivate a "velvet" touch. Be careful to eliminate parallax and obtain a correct focus. If the instrument goes slightly out of the level during one set of horizontal angles do not correct it till the set is completed. This is a very important precaution to note as a defective fitting in any footscrew is quite sufficient to give a horizontal "roll" and vitiated horizontal readings. In *vertical* angle work satisfy yourself thoroughly as to the levelment of the vertical arc bubble, or the

bubble on the telescope as the case may be, and correct if necessary at each observation by antagonising screw or either screws if there is no further necessity for horizontal angular work, or apply bubble correction, *vide* Appendix VII.

Intersected points are observed in the same manner as stations, but only one set of angles is necessary or one zero and one vernier (invariably A vernier).

The description of the point intersected should be clearly stated, as it is not always that the triangulator subsequently does the plane-tabling, so that the man who follows should have no doubt as to which point is meant and to what position of the point a height has been given. For example, church spires and temples having lightning conductors the horizontal angles would be taken to the lightning conductors, but vertical angles would be taken to some point easily recognisable directly beneath it—this should be carefully described. Again, a tree would be observed to for horizontal angles as near the ground as possible, but if the ground line were not visible it would be best for vertical angles to be taken to the highest part of the tree, though as a rule where the ground line is seen it should always be taken and sometimes both ground line and top values recorded. Sketches of objects are very useful, and these are best made by drawing them in the book when held upside down, for this reason that the object in the telescope is inverted, and if drawn as seen through the telescope with the book reversed, the sketch will be right way up when the book is held in its normal position.

In describing trees it is not of much use describing their colour, much better to try and make out what kind of tree it is. Such trees as mangos and tamarinds are easily recognisable, but if in doubt refer to a local man and he will generally give you the correct name. It is best not to classify the tree at all if there is a chance of it being misnamed. A good practice in describing trees is to place them north, south, east or west of a group or a pair of trees, etc. Avoid writing "right" or "left" of an object as this depends entirely on the position from which it is viewed.

Form C.

Magnetic direction of Zero Station 191° 31′ 00″.

Angles observed at A. Horizontal Angles. *Cooke T. and S. Transit Instrument No.* 4377 (5″), *date* 22-1-13.

Station.	R 0. (Right Swing) A. ° ′ ″	B ′ ″	Mean. ° ′ ″	L 180. (Left Swing). A. ° ′ ″	B. ′ ″	Mean. ° ′ ″	R 90. (Right Swing). A. ° ′ ″	B. ′ ″	Mean. ° ′ ″	L 270. (Left Swing). A. ° ′ ″	B. ′ ″	Mean. ° ′ ″	General Mean. ° ′ ″
Station B ...	0 01 00	01 00	0 0 0	180 00 40	01 00	0 0 0	90 01 20	01 20	0 0 0	270 01 20	01 20	0 0 0	0 0 0
,, C ...	306 10 20	10 40	306 09 35	126 10 20	10 40	306 09 50	36 11 00	11 00	306 09 40	216 10 40	11 00	306 09 30	306 09 39
,, D ...	326 33 10	33 20	326 32 35	146 33 00	33 20	326 32 30	86 33 40	33 40	326 32 20	236 33 40	34 20	326 32 40	326 32 26
,, B ...	0 00 40	01 00	0 0 0	180 00 20	00 40	0 0 0	90 01 20	01 20	0 0 0	270 01 00	01 40	0 0 0	0 0 0

Vertical Angles.

Station.	E or D.	R. A. ° ′ ″	R. B ′ ″	R. Mean ° ′ ″	L. A. ° ′ ″	L. B. ′ ″	L. Mean. ° ′ ″	E or D.	General Mean. ° ′ ″	Height of signal observed. ′ ″	Remarks and description of station.
Station B ...	D	0 00 20	00 40	0 00 30	0 00 30	00 40	0 00 30	D	0 00 30	9 5	Station mark is an embedded brick on the left bank of the Birampur distributary canal about 1,100 feet S. of a syphon over which a cart track from Birampur to Thoi village passes. Birampur village lies roughly 500 yards to the S.W. The mark is 10 feet offset from the water's edge. The mark is superimposed by a small pillar and instructions have been left that there shall be an upper mark exactly 2 feet above original mark-stone.
,, C ...	D	0 06 20	06 20	0 06 20	0 06 20	06 40	0 06 30	D	0 06 25	9 4	
,, D ...	D	0 00 20	00 20	0 00 20	0 00 00	0 20	0 00 10	D	0 00 15	10 1	

Height of signal 8′ 9″. Height of instrument 5′ 2″.

Form C.

Magnetic direction of Zero Station 191° 31′00″.

Angles observed at A. HORIZONTAL ANGLES. *Cooke T. and S. Transit Instrument No.* 4377 (5″), *date* 22-1-13.

Station.	R 0. (Right Swing). A.	R 0. B.	R 0. Mean.	R. 180. (Left Swing). A.	R. 180. B.	R. 180. Mean	R. 90 (Right Swing) A.	R. 90 B.	R. 90 Mean.	L. 270. (Left Swing). A.	L. 270. B.	L. 270. Mean	General Mean
	° ′ ″	′ ″	° ′ ″	° ′ ″	′ ″	° ′ ″	° ′ ″	′ ″	° ′ ″	° ′ ″	′ ″	° ′ ″	° ′ ″
Station B ...	0 01 20	... ...	0 0 0	180 01 00	... ...	0 0 0	... ...	... ...	... ...	... ...	... ...	... ...	0 0 0
Tall palm (1) ..	257 52 00	... ...	257 50 40	77 51 20	... ...	257 50 20	... ...	... ...	... ...	... ...	... ...	... ...	257 50 30
Shisham tree brush (2)	328 27 00	... ...	328 25 40	148 26 20	... ...	328 25 20	... ...	... ...	... ...	... ...	... ...	... ...	328 25 30
Little green bush brush (3)	338 44 00	... ...	388 43 00	158 43 40	... ...	338 42 40	... ...	... ...	... ...	... ...	... ...	... ...	338 42 50
etc. ...	... ...	... ...	... ...	... ...	... ...	... ...	... ...	... ...	... ...	... ...	... ...	... ...	... ...
etc. ...	... ...	... ...	... ...	... ...	... ...	... ...	... ...	... ...	... ...	... ...	... ...	... ...	... ...
Station B ...	0 01 20	... ...	0 0 0	180 01 00	... ...	0 0 0	... ...	... ...	... ...	... ...	... ...	... ...	0 0 0

VERTICAL ANGLES.

Station.	E or D	R A.	R B.	R Mean.	L. A.	L. B	L. Mean.	E or D	General Mean.	Height of signal observed.	Remarks and description of station
		° ′ ″	′ ″	° ′ ″	° ′ ″	′ ″	° ′ ″				
(1)	D	0 24 40	... ...	... ...	0 23 40	... ...	... ...	D	0 23 50	...	To ground level.—In cultivation, single and conspicuous, about ½ mile N.E. of Birampur.
(2)	E	0 03 40	... ...	... ...	0 .24 ~~40~~ 00	... ...	... ...	E	0 03 50	...	To top of tree.—In centre bush jungle, on slight eminence close to main stream.
(3)	D	0 05 20	... ...	... ...	0 06 20	... ...	... ...	D	0 05 50	...	To top of tree.—In-bush jungle on a conspicuous knoll about 700 yards E. of Birampur village and 20 yards W. of main stream. Cart track and footpath from Birampur to Nurnagar passes 30 yards to the S. of bush.

Height of signal 8′ 9″. Height of instrument 5′ 2″.

8. So that computations of a quadrilateral figure (*see* Plate I.) with a supplementary station and a Satellite or eccentric station involved, can be followed, abstract angles from the field-book of an actual piece of work are here given with the full computations to follow in the respective forms for the purpose. The base line AB in this particular case was along the almost level bank of a canal, and its reduction was a simple matter and does not appear in the form suggested for that purpose. The azimuth or direction from true N of the station C was observed at station A by taking an ex-meridian observation to the sun.

Stations.	Horizontal angles.			Vertical angles.			Height of instrument.		Height of signal.		Remarks.
	°	′	″	°	′	″	′	″	′	″	
At A to station B	0	0	0	D 0	00	30	5	2	9	5	To signal.
,, ,, C	306	09	39	D 0	06	25	...		9	4	,, ,,
,, ,, D	329	32	26	D 0	00	15	...		10	1	,, ,,
,, Supplementary	323	58	56	...	...	...	...		...		
At B to station A	0	0	0	E 0	10	36	5	1½	9	5	To signal.
,, ,, C	85	05	46	D 0	02	50	...		...		,, ,,
,, ,, D	120	24	01	E 0	04	40	..		.		To ground level.
To tall palm ...	20	56	35	D 0	01	20	...		...		
At C to station D	0	0	0	E 0	19	10	5	1½	9	4	To signal.
,, ,, B	73	58	04	E 0	10	55	...		...		,, ,,
,, ,, A	115	01	47	D 0	11	55	...		...		
At D to station B	0	0	0	E 0	04	05	5	3	10	1	To signal.
,, ,, A	26	08	31	E 0	04	55	...		...		,, ,,
,, ,, C	70	43	41	D 0	04	25	...		...		
At supplementary S to station B	0	0	0	E 0	25	37	4	10	8	10½	To signal.
,, ,, A	54	47	24	E 0	19	00	...		...		,, ,,
,, ,, C	170	09	29	E 0	19	30	...		...		,, ,,
,, ,, D	241	36	06	E 0	25	55	...		...		To ground level.
To tall palm ...	75	09	20	E 0	50	00	...		...		

9. By trigonometry, if ABC is a triangle then $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ and with the angles A,B,C known also the side A B (c) measured, then $a = \frac{\sin A}{\sin C} \cdot c$; and $b = \frac{\sin B}{\sin C} c$,

or $\log a = \log c + \log \sin A + \log \operatorname{cosec} C$ etc.

10. We shall proceed to put this formula into a convenient form which is Form D for calculation of horizontal distances with the following instructions :—

First examine the ~~plane-table~~ reconnaissance ~~plot~~ and choose the best conditioned triangles and aim at obtaining a double value for a side on which an extension is being made. The example given in reconnaissance plot is of a quadrilateral with A B as the known base, and it is evident that CD is the base on which the next quadrilateral will be built. The triangles are to be written *anti-clockwise* taking the known side first. In example, ABC will be the first triangle to be solved of which the side AB and the three angles are known. It is to be written as ABC. The anti-clockwise direction is selected as the bearings are from North and the interior angles of the figure thus become inward angles (*see* Chapter V. on Traversing, Part I).

The interior angles of each triangle must equal 180° 0′ 0″, and any error must be distributed *equally* in whole seconds and not proportionally to the size of angles—any residue again equally to the larger angles. In this case of minor triangulation no attempt need be made to grind down the angles or to obtain the probable errors by least squares, etc.

Having obtained the corrected angles take out the log sines for the first and second angles and the log cosec for the third, and add the logs of the first and third to the given log base to obtain the log side of the first, and add the logs of the second and third to the given log base to obtain the log side of the second.

N.B.—The reader will notice that, by holding a pencil over one line, cols. log sines and log feet, there remains in view the correct values to add and the space into which the result is to be placed.

Finally take out the antilog for "feet" and complete form.

Where double values or common sides exist their mean must be taken and entered in each triangle and accepted for further extension work if used as a base. The last two triangles worked out in example are for an intersected point, and the third angle in each triangle is therefore supplementary. These two triangles give a common side and without this check an intersected point is doubtful. Triangles should be properly numbered.

The form offers the following advantages :—It is compact and the side appears opposite or on the same line as the angle, that is, *a* or BC appears on the same line as angle A.

Form D.

△ 1. *Log feet* = 3 · 445 9559 *from* △ *base* = 2792·26′.

Stations	Angles observed °	′	″	Apport of error ′	″	Reduced angle for computation °	′	″	Log sines.			Log feet			Feet.	Side.
A	53	50	21	+	3	58	59	24	$\bar{1}$	907	0739	3	535	5401	3431·95	B C.
B	85	05	46	+	4	85	05	50	$\bar{1}$	998	4081	3	626	8743	4235·20	A C.
C	41	03	43	+	3	41	03	46	0	182	5103					
	179	59	50	+	10	180	00	00								

△ 2. *Log feet* = 3 · 445 9559 *from* △ *base.*

A	33	27	34	−	2	33	27	32	$\bar{1}$	741	4183	3	543	3418	3494·15	B D.
B	120	24	01	−	2	120	23	59	$\bar{1}$	935	7672	3	737	6907	5466 27	A D.
D	26	08	31	−	2	26	08	29	0	355	9676	3	543	3476	3494·2(	Mean.
	180	00	06	−	6	180	00	00	..	...	..	3	737	6938	5466·31	Mean.

△ 3. *Log feet* = 3 626 8743 *from* △ 1

C	115	01	47	+	4	115	01	51	$\bar{1}$	957	1667	3	737	6968	5466·34	A D.
A	20	22	47	+	3	20	22	50	$\bar{1}$	541	8960	3	322	4261	2101·00	C D.
D	44	35	15	+	4	44	35	19	0	153	6559	3	737	6938	5466 31	Mean.
	179	59	49	+	11	180	00	09								

△ 4. *Log feet* = 3 · 535 5401 *from* △ 1

C	73	58	04	−	2	73	58	02	$\bar{1}$	982	7703	3	543	3534	3494 24	
B	35	18	15	−	1	35	18	14	$\bar{1}$	761	8625	3	322	4456	2101·11	
D	70	43	46	−	2	70	43	44	0	025	0430	3	543	3476	3494 22	Mean.
	180	100	05	−	5	180	00	00	...			3	322	4359	2101 05	Mean.

△ 56. *Log feet* = 3 · 445 9559 *from* △ *base*

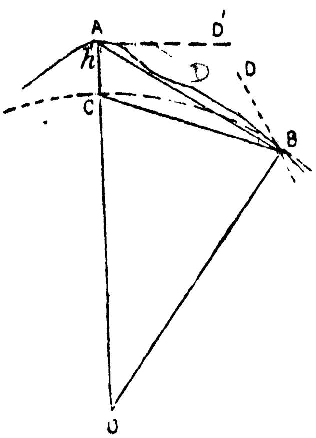
A	102	09	30	..	..	...	...	...	$\bar{1}$	990	1476	3	513	0121	...	...
B	20	56	35	...	...	...	.		$\bar{1}$	553	2031	3	076	0676	1191·37	...
Tall palm	Suppl			.	...	56	53	55	0	076	9086	...	...	...	...	...
	..	...	...	...	.	180	00	00	...	..	...	3	076	(790	1191·38	Mean.

△ 57. *Log feet* = 3 · 533 6924 *from* △ *Form F.*

A	66	08	26	...	...	...	...	..	$\bar{1}$	961	2029	3	495	7033	...	...
Sappl. S	20	21	56	..	...	...	.	...	$\bar{1}$	541	5899	3	076	0903	1191·39	...
Tall palm	Suppl.			...	.	93	29	38	0	000	8080	...	...	..	...	...
				...	...	180	00	00	...	.	...	3	076	0790	1191 38	Mean.

11. **Computation of heights** (*see* also Appendix V.)—Let A and B be two points on the earth's surface and O the centre of the earth. AD′ and BD perpendiculars to AO and BO respectively will represent the directions of a level surface at A and B respectively, and the angle BAD′ will sensibly equal to the angle ABD if the reciprocal angles D′AB and DBA had been simultaneously observed when the refraction would have been equal or constant.

Fig. 3.



Make OC = OB, then AC = h or the difference of height of A and B.

Let the observed depressions at A and B be D_a and D_b. Now angle ABC = angle OCB − angle BAO, and angle ABC = angle ABO − angle CBO; and since angle OCB = angle CBO, then angle ABC = $\frac{1}{2}$ (angle ABO − angle BAO); also since angle BAD′ = angle ABD, then angle ABC = $\frac{1}{2}$ (angle DBO − angle D′AO) = $\frac{1}{2}(D_b - D_a)$ = S.

Now BC is a very small part of the earth's surface. So that h may be put equal to BC tan ABC or h = BC tan $\frac{D_b - D_a}{2}$, or if one angle is an elevation h = BC tan $\frac{E_a + D_b}{2}$, or BC tan $\frac{E_b + D_a}{2}$, that is, subtended angle S is equal to half the algebraic difference of two depressions or two elevations, and is equal to half the algebraic sum of a depression and an elevation with the proper sign. If one angle has only been observed from figure, it will be seen that if the refraction is R* for angle BAD′ then R = $\frac{BC - D_a - D_b}{2}$; but when reciprocal angles have been observed the value R can be found and $\frac{R}{BC}$ is called the *co-efficient of refraction* which in India may be considered as about ·067.

Now h in the above formulæ represents the difference of heights between the ground levels of A and B, and if i_a = height of instrument at A, g_a = height of signal at A, i_b height of instrument at B, g_b height of signal at B, and if A is the station the height of which is known, and B be the station the height of which is required, then height of B = height of A ± BC tan S + $\frac{g_a - g_b + i_a - i_b}{2}$.

If one angle only has been observed then a correction for curvature and

*Appendix 2 Table or as a rough rule:—Correction (for Curvature and Refraction) in feet = $\frac{4}{7}$ of the square of the distance between the two stations in statute miles.

refraction must be considered and this correction is always plus and is obtained from Table (*see* Appendix II.). The table gives the angle for the log of the subtended side. The algebraic sum is the value of S.

Refraction is more or less constant between the hours of 1 and 3 P. M. and as simultaneous ~~observations~~ cannot be taken, vertical angles are best observed between the above hours.

Form E has been drawn up to accommodate both cases (1) for reciprocal values (2) for single values. The log side is obtained from Form D.

As a word of caution the student is warned that no great reliance can be placed on single value heights, as refraction and pressure changes also the presence of a grazing ray—all tend to vitiate the results.

Also that the heights, derived by reciprocal values, although they seem to accord, are by no means as reliable as heights derived by ordinary levelling, except over very broken and hilly ground, where levelling is almost an impossibility.

Form E.

Known station (A) Height of known station. Station required (B)	A 1015·04 B			A 1015·04 Tall palm.			B 1010 22 Tall palm					
	△ No. 1.			△ No			△ No			△ No.		
(1) *For reciprocal values.*	°	′	″	°	′	″	°	′	″	°	′	″
+ or − D_a ...	−0	00	30									
+ or − E_b ..	+0	10	36									
Algebraic sum = 2S ...	−0	11	06									
S* ..	−0	05	33									
(2). *For single values*												
$\pm\frac{E_a}{D_a}$				−0	23	50	−0	04	20			
Correction for C & R (*see* Table)†	+			+		05	+		14			
Algebraic sum = S				−0	23	45	−	04	06			
Log tan S	$\bar{3}$	208	0195	$\bar{3}$	839	39	$\bar{3}$	076	51			
Log side	3	445	9558	3	076	08	3	513	01			
Log h	0	653	9753	0	905	47	0	589	52			
h ±	−4·51			−8·42			−3·9					
(1). $\frac{g_a - g_b + i_a - i_b}{2}$ (2) $\pm (i_a - g_b)$	−0·31			+5·1			+5·1					
Known height of A ...	1015·04			1015·0			1010·2					
Deduced height of B ...	1010·22			1011·9			1011·4					
Mean value ...				(Mean).			1011·7					

Where E_a = Elevation at A, g_a = height of signal at A.

D_a = Depression at A. g_b = height of signal at B.

E_b = Elevation at B. i_a = height of instrument at A.

D_b = Depression at B. i_b = height of instrument at B.

* Appendix IV. for calculation of logs of a small angle.

† Appendix 2.

12. **Form F** is for the computation of a point from which observations have been taken to three known points, in other words, it is the solving of the plane-table fixing by trigonometry. Here all the angles of the triangle ABC are known, also the angles a and β observed from S the point required to be found. It is evident that the angles CAS and CBS are required. Call these angles x and y respectively.

Then $C + a + \beta + x + y = 360°$

Let $(C + a + \beta) = P$.

Then $x + y = 360 - P$.

And $CS = \frac{b \sin x}{\sin a} = \frac{a \sin y}{\sin \beta}$.

By trigonometry $\frac{\sin a}{\sin \beta} \times \frac{a}{b} = \frac{\sin x}{\sin y}$.

$\therefore \frac{a \sin a}{b \sin \beta} = \frac{\sin x}{\sin y} = \tan \phi$ say

By adding and subtracting—

$$\frac{\sin x - \sin y}{\sin x + \sin y} = \frac{\tan \phi - 1}{\tan \phi + 1} = \tan (\phi - 45°),$$

$$\text{but } \frac{\sin x - \sin y}{\sin x + \sin y} = \frac{\tan \frac{1}{2} (x - y)}{\tan \frac{1}{2} (x + y)}.$$

$$\therefore \tan (\phi - 45°) = \frac{\tan \frac{1}{2} (x - y)}{\tan \frac{1}{2} (x + y)}$$

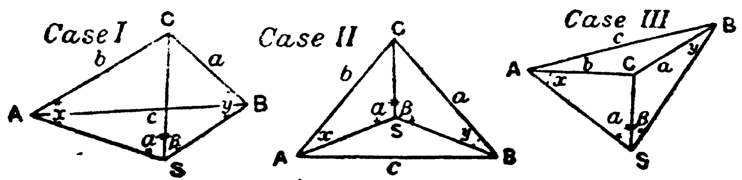
$$\text{or } \tan \tfrac{1}{2} (x - y) = \tan \tfrac{1}{2} (x + y) \times \tan (\phi - 45°)$$

and this has been adapted to logarithmic computation in a convenient way. The right hand portion of the form is only a repetition of Form D for calculation of sides. Notice that the angle ABC is supplementary. The sum of ACS and BCS should equal the angle ACB, but this rarely exactly happens, chiefly because the functions of three triangles are employed to obtain its value. An example is worked out from the values given in the abstract contained in para. 8.

Supplementary stations should be made more use of than they are as they frequently aid in filling a void or where points are few and by intersection and computation many intersected points by two rays can be finally fixed by a third from a supplementary station. Such stations can easily be observed at on a line of march and need not be on a hill top but preferably where, as mentioned before, points are lacking. The author tried whenever possible, to set up near the margin of four planetable sections so that the point fixed may give work in four plots.

Form F.

	Angles. ° ′ ″		Logarithms.	Stations.	Angles. ° ′ ″	Log Sines.	Log feet.	Feet.	Side.
*C (or 360°—C)... (see △ 3, page 15).	115 01 51				Log b =	3 626 8743			
a ...	115 22 05	Log sin a ...	$\bar{1}$ 955 9639	A (x)	17 49 18	$\bar{1}$ 485 8000	3 156 7104	1434·47	CS.
β ...	71 26 37	Log cosec β ..	0 023 1867	C (sup)	46 48 37	$\bar{1}$ 862 7820	3 533 6924	3417·37	AS.
Sum = P ...	301 50 33	Log a ...	3 322 4359	S (a)	115 22 05	0 044 0361			
360° − P = $x + y$	58 09 27	Ar. Co. Log b ...	$\bar{4}$ 373 1257		180 0 0		3 156 7094	1434·47	
ϕ ...	25 18 24	Log tan ϕ ...	$\bar{1}$ 674 7122		Log a =	3 322 4359			
$\phi - 45°$	−19 41 36	Log tan (ϕ − 45)	$\bar{1}$ 553 7867	B (y)	40 20 10	$\bar{1}$ 811 0857	3 156 7083	1434·48	CS.
‡ $\frac{x+y}{2}$	29 04 44	Log tan $\frac{x+y}{2}$ ...	$\bar{1}$ 745 1610	C (sup) S (β)	68 13 13 71 26 37	$\bar{1}$ 967 8367 0 023 1867	3 313 45[illegible]	2058 06	BS.
† $\frac{x-y}{2}$	−11 15 26	Log tan $\frac{x-y}{2}$ ...	$\bar{1}$ 298 9477		180 00 00		3 156 70[illegible]4	1434·47	Mean.
x ...	17 49 18								
y ...	40 20 10								



* In case III, 360 − C must be used instead of C.

† When $\frac{x+y}{2} < 90°$, $\frac{x-y}{2}$ has the same sign as $\phi - 45°$.

When $\frac{x+y}{2} > 90°$, $\frac{x-y}{2}$ has the opposite sign to $\phi - 45°$.

‡ The sign for $\frac{x+y}{2}$ is always +.

If C + α + β = 180°, the problem is insoluble.

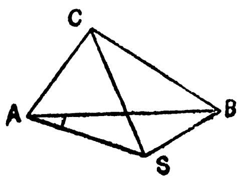
Note.—In the above it will be seen that the angle at A between C and supplementary S is 17° 49′ 18″ with which compare the same angle as actually observed (for purposes of illustration only) in the abstract given in para. 8, *viz*, 17° 49′ 17″.

18. **Form G.**—It sometimes happens that the only desirable position on a hill top is occupied by a tomb, pagoda or other permanent structure, and that a station on the particular hill is a necessity for the continuation of the triangulation, or for the purpose of taking 3rd intersections to fix certain intersected points which are sure otherwise to become obscured. Most manuals give an example or explanation of what is known in India as a Satellite station, and in America as an Eccentric station, and one is led away with the idea that a Satellite station is of common occurrence. This is not so. With the best laid out triangulation it is never used, and with tertiary or minor triangulation its occurrence is rare as it can so easily be avoided by the exercise of a little forethought or by a supplementary station as in previous para.

The most likely circumstance in which it will occur is as noticed for the reintersection of some doubtful points. For instance in a city survey if a flag-staff on a tower had been intersected from stations, and it was found necessary to make a station somewhere on the tower, the position directly beneath the staff being an impossible one, the theodolite is set up in some convenient position, say S (in the diagram for Case I., Form F), B being the flag-staff and A and C the stations from which B was observed.

At S the instrument is set to Magnetic North and the readings to A, C and B are taken with the same care as at stations ; the Magnetic North being the zero station, and the lower plate is clamped to this direction. S B is also carefully measured with a tape.

To reduce the angles at S to angles at B we proceed as follows :— In the triangle ACB the angles at A and C are known and the base AC ; from these data we can obtain a very nearly correct value for the sides AB and BC. In the triangle ASB the angle ASB (S) has been observed, SB (a) has been measured and AB (s) is known, and if we consider the value of the angle SAB in seconds we find it equals $\frac{\text{SB} \sin \text{ASB}}{\text{AB} \sin 1''}$ in which $\frac{\text{SB}}{\sin 1''}$ is a constant.



In the triangle ASB we have therefore—

$$\text{A (in secs)} = \frac{a}{\sin 1''} \times \frac{\sin \text{S}}{s}$$

$$\therefore \log \text{A}'' \doteq \log a + \log \sin \text{S} - \log s - \log \sin 1''$$

where $\log \sin 1'' = 6{\cdot}6855749$.

This gives us the correction to be applied to the magnetic direction of SA to obtain BA, and similarly of SC to obtain BC ; by subtracting

the value obtained of BC from BA we get the included angle ABC, and obviously the magnetic directions have no significance after the deduction has been completed. The sign of sin S must be carefully attended to, and to avoid all chance of mistakes in this respect it is as well that a diagram with the direction N is drawn and the sign applied accordingly. In the example, if CA was pointing due N then the bearing of SA would be greater than BA and the angle SAB would be subtracted from the direction SA to obtain the direction BA.

For the purpose of illustration an example is worked out as follows:—

Let it be supposed that the theodolite could not be set up over station mark C, and the following observations were made at a station point C′ = 129′ 4″ from C. The angles reduced from C′ to C should be compared with the actual observations taken at C, (*see* also Plate 1.)

At C′ Magnetic North	..	...	...	0	0	0
to Station D	...	.	...	204	05	58
" " B	...	...	...	276	54	36
" " A	...	...	...	316	38	56
" " C	...	...	.	329	25	26

Form G.

$$\text{Log } \frac{a}{\sin 1''} = \text{constant log} = 10 + 2{\cdot}1116993 - 4{\cdot}6855749 = 7{\cdot}4261244.^{*}$$

Station.	Data	Logarithmic computation.		Corrections.
	Constant Log ...	7	4261244	
	Log sin B (angle BC′C)	$\bar{1}$	8995474	
	Total ...	7	3256718	Direction B to C′ = 276° 54′ 36″.
	Log *s* (B C) ...	3	5355401	
	Log A″ ...	3	7901317	Angle C′BC = 1° 42′ 47″.
	A = ...		6167″ = 1° 42′ 47″	∴ Direction C to B = 275° 1′ 49″. †Included angle BCA = 41° 03′ 54″.
	Constant Log .	7	4261244	Direction A to C′ = 316° 38′ 56″.
	Log sin S, (angle AC′C)	$\bar{1}$	3446339	Angle AC′A = 23 13.
	Total ...	6	7707583	∴ Direction C to A = 316 15 43.
	Log *s* (A C) ...	3	6268743	†Included angle BCA = 41° 03′ 54″.
	Log A″ ..	3	1438840	
	A = .		1393″ = 23′ 13″	

* $\frac{\sin S}{s} = \frac{\sin SAB}{a}$; but as angle SAB is very small sin SAB = number of seconds in SAB × sin 1″; therefore if A = number of seconds in SAB then $\frac{\sin S}{s} = \frac{A \times \sin 1''}{a}$ or $A = \frac{a}{\sin 1''} \times \frac{\sin S}{s}$, that is, $\frac{SB}{\sin 1''}$ is a constant.

† This angle is obtained by subtracting the direction as deduced from one triangle from the direction deduced in the other—the resulting value to be entered in both compartments of the form Compare angle at station O between A and B (para 8).

It sometimes happens in a round of angles at station that unfortunately one station, to which observations are necessary, is obscured, in which case the instrument is removed to a position where the station is visible, and with the same procedure as given above the angle is reduced to the original mark.

The reduction of angles of Satellite stations is accurate, but a laborious process, and for a short series of the tertiary order, such as an engineer would run, it would not be wrong to advise that the station be made a supplementary station and computed as such as shown in Form F.

14. **Computation of the third side from two sides and the included angle.**—As explained in a previous para. in this chapter, it sometimes happens that a side is required for extension of triangulation, or that in the extension of such triangulation from a measured base it is required to test the accuracy of it. In example, we are to suppose that no reciprocal observations were taken between A and D, and that we wish to test the accuracy of the extension from the base line AB, that is, we wish to test the distance A to D.

The following formulæ are utilised in the form:—

For Form H *see* page 23.

$$\text{Tan}\,\frac{A-B}{2}=\frac{a-b}{a+b}\cot\frac{C}{2}\quad\text{and } c=(a+b)\sin\frac{C}{2}\sec\frac{A-B}{2}$$

the mean of the two values for AD should be accepted. Compare the values with the values given in triangles 2 and 3 of Form D.

The triangulation is now set up and balanced by using the Traverse Form I., and the quadrilateral ABDC has been computed as a guide, and some remarks, already made under Chapter on Traversing, (Part I.) are repeated.

15. **Form I.**—*Computation of rectangular co-ordinates.* This form is best explained by taking the example in which A is considered the Origin and the bearing of A to B, or the line AB is given as 192° 12′ 54″. The circuit, in this case, ABDCA, is taken *anti-clockwise* because in traversing the *inward* angles are observed, and this is a closed circuit as the starting and ending station is one and the same. In a closed circuit the angles are corrected to add up to a certain sum by Euclid 1. 32, Cor. 1., and in a long traversed line by a check azimuth to which the correction for convergency has been applied to the azimuth observed. In computing co-ordinates of triangulation the angles are entered to seconds, but in traversing, angles to the nearest minute only are necessary, except for city work on large scales when the angles may be entered to the nearest whole reading of a vernier division.

Form H.

CALCULATION OF THE THIRD SIDE FROM TWO SIDES AND INCLUDED ANGLE.

Triangles.	Data.	Formula $\tan\frac{A-B}{2} = \frac{a-b}{a+b}\cot\frac{C}{2}$	Logarithmic Computation.		Value of $\frac{A-B}{2}$	Formula $c = (a+b)\sin\frac{C}{2}\sec\frac{A-B}{2}$	Logarithmic Computation.		Length of third side in feet.	
ABD	BD = 3494·20	BD + AB = 6286·46	Ar. co	$\bar{4}$·2015939		BD + AB = 6286·46	Log	3·7984061		
	AB = 2792·26	BD − AB = 701·94	Log	2·8463000			Log sin	$\bar{1}$·9384024		
	∠ABD = 120° 23′ 59″	$\frac{C}{2}$ = 60° 12′ 00″	Log cot	$\bar{1}$·7579313			Log sec	0·0009024		
			Log tan	$\bar{2}$·8058252	3° 39′ 32″		Log	3·7377109	5466·52	AD
ACD	CD = 2101·05	AC + CD = 6336·25	Ar. co	$\bar{4}$·1981676		AC + CD = 6336·25	Log	3·8018324		
	AC = 4235·20	AC − CD = 2134·15	Log	3·3282250			Log sin	$\bar{1}$·9261042		
	∠ACD = 115° 01′ 51″	$\frac{C}{2}$ = 57° 30′ 56″	Log cot	$\bar{1}$·8039271			Log sec	0·0097191		
			Log tan	$\bar{1}$·3303197	12° 04′ 35″		Log	3·737 557	5465·83	AD

In the 3rd column the values are all *bearings*, that is, they are directions to a fixed meridian on the earth's surface; if therefore, any azimuth is observed at some distance E. or W. of the meridian of the origin, and a direction found with respect to a celestial object for angular check or the traverse is tied to a triangulated station—the difference of convergency of meridians must be applied to the azimuths to convert them into bearings

In the Northern Hemisphere convergency is $\pm$ according to whether the departure is $\frac{\text{West}}{\text{East}}$ and *vice versâ* in the Southern Hemisphere. Convergency may be deduced as follows:—Add to the constant log in feet (4·2162) log tan latitude (obtained roughly from the standard map), also log departure in feet (taken from the reconnaissance plot) and the result will be the log convergency in minutes. In latitude 30° the convergency is almost exactly $\frac{1}{2}$ minute per mile, or $\bar{4}\cdot2164 + \bar{1}\cdot7614 + 3\cdot7226 = \bar{1}\cdot7004 = \frac{1}{2}$ minute.

The logs in the calculations for triangulation, if the sides are long, should be taken out to 7 figures. Having found the distances on the meridian and perpendicular they are entered in the columns for that purpose with reference to the quadrants of the bearings. To *balance* the traverse these columns are totalled and the difference Northing and Southing also Easting and Westing should be equalised in a closed circuit, or in other words, since the traversing returns to the origin the departure "Westing" must equal departure "Easting" and the latitude "Northing" must equal the latitude "Southing". Let there be an error of + ·23 foot. This quantity in a closed circuit must be halved and ·12 foot added to the lesser and ·11 foot subtracted from the greater total, and then the ·12 foot distributed proportionally in the several values which go to make up the total. In a traverse which is not a closed circuit the starting and closing co-ordinates being known the difference in the latitude and departure is therefore known, and for the traverse to *close* the difference in the total latitudes and departures of the traverse must equal the differences in the given co-ordinates, if not, the error must be distributed as explained.

The last two examples in the form are to find the co-ordinate values of the supplementary station and an intersected point; the area, by successive ordinates multiplied by distances on the meridian, has been explained under traversing, Chapter V., Part I.

Form I. TRAVERSE FORM.

1	2	2A	3	4	5	6	7		8		9		10		11	12	13	14	15	16
							Distances on the													
							Meridian (Latitude)				Perpendicular (Departure).									
Station.	Inward Angles.	Correction.	Bearing	Reduced Bearing	Distance (feet)	Cardinal Direction	N.	Cor	S	Cor.	E.	Cor.	W	Cor.	Latitude. N or S.	Departure E. o W	Successive Ordinates	Double Area. +	−	Remarks.
	° ′ ″	″	° ′ ″	° ′ ″				(a) −		(a) +		(a) −		(a) +						
															0	0				
A	120 23 59	..	192 12 54	12 12 54	2792 26	SW	.	..	2728·87	07	.	..	590 79	..	S 2728 94	W 590·79	W 590 79	1612226		
L	70 43 44	1	132 36 53	47 23 07	3494 20	SE		..	2365·80	03	2571·46	03	..	..	S 5094 89	E 1980 64	E 1389 85		3288177	
D	115 01 51	.	23 20 38	23 20 38	2161 05	NE	1929 06	04	..	.	832·54	..	.	.	S 3165·87	E 2813 18	E 4793 82	9247372		
O	53 50 24	1	318 22 29	41 37 31	4235 20	NW	3165 84	07	...	.	.	.	2813·14	·04	0	0	E 2813 18	8905890		
A														·04						
		+			12622 71		5094 80	11	5094·67	12	3404 00	·03	8403 93					19765488	3288177	
B	359 59 58	2	192 12 54				5094 67				3403·93							3288177		
							23				·07							16477311		
																	2	8238656	sq ft =	$\frac{8238656}{43560}$ acres.
																			=	183·82 acres
									+		−									
															0	0				
C			318 22 29																	
A	17 49 18	..	156 11 47	23 48 13	3417·37	SE	...		3126,67	·04	1379·26	04	...	...	S 3126·71	E 1379·22				
S	115 22 05	.	91 33 52	88 26 08	1434·48	SE	...		39,16		1434 01	05		...	S 3165 87	E 2813·18				
C	ˏ 48 37	...																		
									3165 83		2813·27									
									3165 87		2813 18									
									·04		09									
								−							0	0				
A	102 09 30	...	270 03 24	89 56 36	...	NW	1 2	·6	...	..	...	...	1191 5							
B	90 56 35	..	192 12 54	...	...	...	..	...	...	...	.	...	...	...	S 2728·94	W 590·79				
Tall palm	56 53 55	...	33 09 29	33 09 29	...	NE	2728·9	6	...	.	1782 2	·1	...	...	S 0·6	E 1191 5				
							2730 1	−			1782 2									
							2728·9	1 2			1191 5									
							1 2				590·7									

Ratio of error $= \sqrt{\frac{(\cdot 23)^2 + (\cdot 07)^2}{12622}} = \frac{1}{52595}$

COMPUTATION FOR CO-ORDINATES.

Station.	A to B	B to D	D to C	C to A	A to Suppl	O to Suppl	A to tall palm	B to tall palm.
Latitude	2728·87	2365·41	1929·06	3165·84	3126 67	39·16	1·2	2728·9
Log Lat	3·4359833	3·3739460	3·2853463	3·5004885	3·4950823	1·5928929	0·07129	3 43582
Log Cos Bearing	$\bar{1}$ 9900274	$\bar{1}$·8305984	$\bar{1}$·9629104	$\bar{1}$·8736142	$\bar{1}$·9613889	$\bar{2}$·4361835	$\bar{4}$·99521	$\bar{1}$·92281
Log Distance	3·4459559	3·5433476	3·3224359	3 6268743	3·5336921	3·1567094	3·07608	3·51301
Log Sin Bearing	$\bar{1}$·32‿‿760	$\bar{1}$·8668596	$\bar{1}$·5979681	$\bar{1}$·8223355	$\bar{1}$·6059543	$\bar{1}$·9998381	$\bar{1}$·99979	$\bar{1}$·73795
Log Departure	2·7714319	3·4102072	2·9204040	3·4492098	3·1396467	3·1565475	3·07607	3·25096
Departure	590·79	2571·62	832 54	2813·14	1379·26	1434·01	1191·5	1782·2

16. **Spherical excess.**—In triangles with an area of 76 square miles and over the three angles will add up to something over and above 180° to be correct, and as a rough rule the spherical excess in seconds is equal to $\frac{\text{area in square miles}}{75\cdot5}$; thus a triangle, the area of which is 76 square miles, has 1″ as spherical excess, and an equilateral triangle of 100-mile sides will have a spherical excess of nearly 1 minute.

17. **Clearing a line.**—When two stations are not intervisible owing to trees and bush jungle which have grown up after a term of years it is necessary to clear the ray. Columns of smoke in the day and flares by night very often fail over long distances to mark the direction, and an accurate direction is necessary in order that more property is not destroyed than is absolutely necessary.

If there is a record of the azimuth of one station to the other then it is necessary only to take an observation on a celestial object to find the meridian passing through one station and to set off the recorded azimuth; but in the event of there being no records and a boundary direct from one point to another is required even though a hill intervenes, then one method is as follows:—

A traverse is run from one station to the other. The co-ordinates are calculated from which is obtained the difference along the meridian (x) and the departure (y), between the stations. The direction of the direct ray between the stations is therefore found as follows:— $\frac{x}{y}$ = cotangent of the angle which the direct line makes with North, and since $\frac{x}{y}$ is known therefore the angle is known which set off from the meridian passing through the station.*

The second method of overcoming the difficulty is as follows:—

On the reconnaissance plot, let CD be the line to be cleared. Select two positions A and B intervisible from each other and from which C and D can be seen. Let the side AB equal unity. Solve the triangle ABC and obtain BC, also solve the triangle ABD and obtain BD, (Form D). Next solve the triangle BCD (Form H) and find the angle BCD. As the angle ACB is known, then direction CD at C can be laid down either with A or B as zero station.

Note.—Compare this solution with two-point problem under Chapter VII., Part I., on Plane-tabling.

18. **A few hints on Triangulation.**—When observing, there must be no parallax (the commonest source of error) that is, on the eye being shifted from side to side for horizontal angles, and up and

* This meridian can be found by a R and L face on Polaris and calculated by Taylor's method *see* Chapter III.

down for vertical angles there must be no "wobble" between the diaphragm wire and the object intersected. If there is a wobble it shows that there still remains some parallax which must be eliminated. As the focus will be infinity once the parallax is eliminated there should be no need to adjust either the eyepiece or object glass.

Avoid observing horizontal and vertical angles at one and the same time. It is doubtful whether the method is any quicker in the end, and it is certain that the horizontal angles suffer in accuracy.

Avoid observing when the atmosphere is "boiling" or quivering as is very often the case in the tropics. Avoid also selecting two sites of stations such that the line joining them just clears or misses a ridge or spur which sets up an intermediate disturbance in the atmosphere. Such lines are known as "grazing rays."

The instrument should be in proper adjustment, and since in triangulation observations are taken to points which differ considerably in elevation the adjustment for the standards, so that the telescope rotates in vertical planes, should be attended to (*see* Chapter III, Part 1).

The stand of the instrument should be thoroughly examined for shake and the nuts tightened, if necessary, before observational work is commenced. The legs of the stand should also be well pressed into the ground, and the observer's stance should be *directly* behind the telescope when intersecting the object and *directly* over the verniers when reading. The readings should be recorded exactly as read.

The centering of the instrument should be most carefully done and also the centering of the signals, remembering that the shorter the sides the greater the error due to non-centering.

For ordinary tertiary triangulation a 5-inch transit instrument reading to 20″ with a good lens is recommended. The stand should, if possible, be fitted with a traversing head or better still the traversing head should be a part of the instrument.

With reference to the lens of a theodolite a good lens permits of a distant object coming into focus and going out of focus with a very slight turn of the screw. When testing a lens and focussing on a distant lightning conductor or spire, the wire, if intersecting the object when in focus, should still intersect it as it goes out of focus. If it does not, it shows that the focussing slide is loose* or has not been properly ground and fitted by the maker. This can only be remedied by an instrument maker; if work must progress then one focus must be used for all observations. In

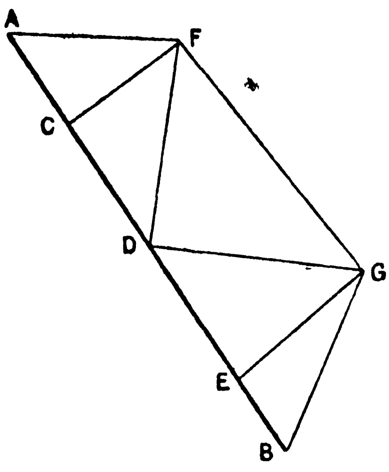
* With the internal focussing lens this error is reduced to a minimum if it exists at all.

triangulation it is rare that the focus needs alteration as the stations are usually all beyond the infinity focus of the telescope.

The footscrews should not be touched during a set of horizontal angles for this reason that if the axis of the footscrew is bent then the whole instrument will have taken on a horizontal movement or roll and the error will be thrown into the *set* of readings. The slight dislevelment of a theodolite will not alter horizontal angles and the observer can satisfy himself by making a test case.

19. **Concerning base lines.**—It is better to measure a fairly short base over an even piece of ground with great accuracy and precision than to measure a longer base under indifferent conditions and with less precision. A base line can be checked by taking points at intervals along the base that is dividing the base into three or more sections setting up a theodolite at two or more points making well-conditioned triangles, observing all the required angles, finally closing on the last section.

Fig. 4.



In figure 4 the theodolite is set up at A, C, D, E, B, F and G and all the angles measured. Accepting AC as part of the whole base AB, and also a base for the triangle ACF the values of CD, DE and EB can be computed by Form D, and hence the whole length of the line AB is found and also separate portions of the line are checked. If the instrument and signals are very carefully centered the check becomes a very efficient one, and will disclose any great discrepancy in any one section of the whole line.

The student should notice here that if ACDE and B are not on one and the same straight line that the direct distance A to B can be accurately found by computing the values of their rectangular co-ordinates, and the differences in departures and along the meridian will give the two sides of a right angle triangle of which AB is the hypotenuse. Similarly, FG the exact distance between two piers of a bridge can be found, and intermediate piers can be located by being aligned between FG, and making certain angles (found by plane trigonometry) at D with either F or G as a zero station.

20. **Concerning mean sea level and base lines.**—All the triangles of a large trigonometrical survey are projected on to the spherical surface of the earth at the level of the base line and then again to the surface at mean sea level (M.S.L.). It is done thus: First the sides of a given triangle are projected on to the horizontal (spherical) plane passing through the lowest angle of the triangle. One of the sides of the given triangle must belong to another triangle, and has therefore already been projected on to the plane passing through the lowest angle of the second triangle. If the second triangle is lower than the first all the sides of the first triangle are projected on to the lower plane, and so on, and eventually the triangulation appears on the terrestial surface at the level of the base line and is then again projected on to the terrestial surface at M.S.L. This reduction is scarcely within the province of engineering surveys (*see* Appendix for reduction to M.S.L.)

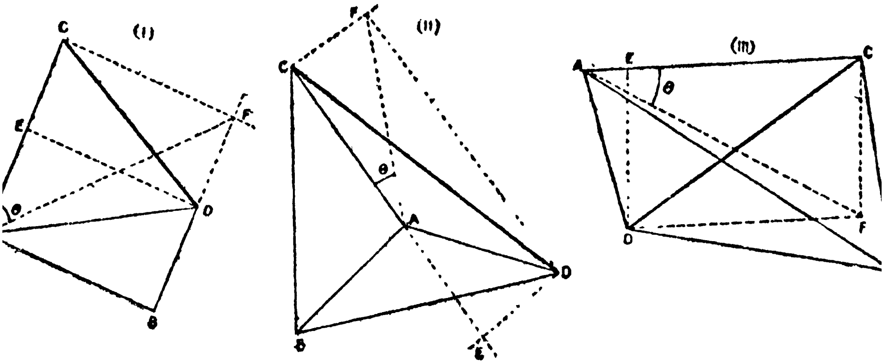
The size of the triangle should never be so large as not to admit of at least two station points being plotted on the size of the sheet of paper according to scale to be fixed on the plane-table. If the reconnaissance sheet (Plate I.) is subdivided up into rectangles or graticules showing the limits, according to scale, of reconnaissance selected for area per sheet or size of plane-table, the triangulation scheme can be mapped out with greater advantage.

A station fixed near the junction of four plane-table sections has the advantage of being utilised four times and is thus of much greater value than one placed in the centre of a plane-table area, though it must not be inferred that stations must be fixed as far as possible near the junction of four sections to the utter exclusion of other situations.

All surveys based on scientific principles which could be accepted by the State for incorporation or correction of the standard maps must be "tied" down or connected to at least two stations or points of the State department work. Since certain points become common to both surveys the State is in a position to judge whether the work is valuable or unreliable for its purposes. An incorrect alignment of a road, railway or canal will only throw suspicion on an otherwise correct map. The little extra care and foresight of connecting work to fixed points of the standard sheets such as Trigonometrically fixed stations, trijunctions of villages fixed by traversing is well worth the trouble, and the professional map-maker will adjust any small discrepancies which may have crept in.

21.—The two-point problem in Triangulation.

Fig. 5.



Problem.—Two stations C and D are known and the distance between. Two other stations or points A and B are intervisible and C and D can be observed to ~~form~~ A and B. It is required to fix the positions of A and B by observations to C and D only.

The above figures represent various configurations of the problem which may be found useful for supplementing points in an area lacking data or on reconnaissance work with an army in the field where connection has been lost between CD and AB and when it is not feasible to return to CD.

Let $CD = d$ and from A are observed the angle BAC and BAD and from B the angles ABC and ABD, and thus all the angles of the triangles ABC and ABD are known.

In the triangle ABD—

$$AD = AB \sin ABD \operatorname{cosec} ADB.$$

and in the triangle ACB.

$$AC = AB \sin ABC \operatorname{cosec} ACB.$$

$$\therefore \frac{AD}{AC} = \sin ABD \sin ACB \operatorname{cosec} ADB \operatorname{cosec} ABC$$

$$= \text{a known quantity.}$$

Thus in the triangle ACD the ratio $\frac{AD}{AC}$ is known, and the included angle CAD is known, so that the triangle can be solved and AC and AD determined.

Let $\frac{AD}{AC} \sin CAD = \tan \theta.$

Draw DE perpendicular to AC or AC produced and complete the rectangle CFDE and join AF.

Then AD sin CAD = DE = CF

$\therefore \tan\theta = \frac{CF}{AC} = \tan CAF.$

$\therefore CAF = \theta.$

$\therefore$ angle DFA = 90° − angle CFA = angle CAF = θ,

and angle DAF = angle CAD — angle CAF = angle CAD − θ

$$\therefore \frac{AD}{DF} = \frac{\sin DFA}{\sin DAF} = \frac{\sin\theta}{\sin(CAD-\theta)} = \sin\theta \operatorname{cosec}(CAD-\theta).$$

$$\therefore \sin CAD \sin\theta \operatorname{cosec}(CAD-\theta) = \frac{AD}{DF}\sin CAD.$$

But DF = CD sin DCF = CD cos ACD.

$$\therefore \sin CAD \sin\theta \operatorname{cosec}(CAD-\theta) = \frac{AD \sin CAD}{CD \cos ACD}.$$

But from the triangle ACD

$$\frac{AD}{CD} = \frac{\sin ACD}{\sin CAD} \therefore AD \sin CAD = CD \sin ACD,$$

whence $\sin CAD \sin\theta \operatorname{cosec}(CAD-\theta) = \frac{CD \sin ACD}{CD \cos ACD} = \tan ACD.$

Thus the angle ACD is found and therefore also ADC; CD is known and the triangle ACD can be solved.

For AC = CD sin ADC cosec CAD,

and AB = AC sin ACB cosec ABC.

The problem is simplified when put into a form suitable for computation.

Example.—The distance CD was given as 21010·4 feet (log 4·3224343) and observations at A and B were made with the following results :—

Angle CAD = 20° 22′ 50″	Angle ABC = 85° 05′ 50″
Angle DAB = 33° 27′ 32″	Angle CBD = 35° 18′ 14″

from which the following supplemental values are deduced, *viz.*,

angle ACB = 41° 05′ 48″ | angle ADB = 26° 08′ 24″

	°	′	″		
Angle ABD	120	24	04	log sin ABD	$\bar{1}$·9357610
,, ACB	41	03	48	log sin ACB	$\bar{1}$·8174945
,, ADB	26	08	24	log cosec ADB	0·3559890
,, ABC	85	05	50	log cosec ABC	0·0015919
				log $\frac{AD}{AC}$	0·1108364
,, CAD	20	22	50	log sin CAD	$\bar{1}$·5418960
,, θ	24	12	15	log tan θ	$\bar{1}$·6527324
				log sin CAD	1 5418960
				log sin θ	$\bar{1}$·6127726
,, θ − CAD	3	49	25	log cosec θ (− CAD)	$\bar{1}$·1759713
,, ACD	115	02	05	log tan ACD	0·3306399

log base 4·3224343.

Station.	Angle.			Log angle.	Log side.
D	70	43	29	$\bar{1}$·9749460	4·5355178
C	73	58	17	$\bar{1}$·9827794	4 5433512
B	35	18	14	0·2381375	
		log base		4·3224343	
D	44	35	05	$\bar{1}$·8463143	4·6268526
C	115	02	05	$\bar{1}$·9571529	4·7376912
A	20	22	50	0·4581040	
		log base		4·5355178	
B	85	05	50	$\bar{1}$·9984081	4·6268551
C	41	03	48	$\bar{1}$·8174945	4·4459415
A	53	50	22	0 0929292	
				4·5433512	
B	120	24	04	$\bar{1}$·9357610	4·7376939
D	26	08	24	$\bar{1}$·6440110	4·4459439
A	33	27	32	0·2585817	

From the above the following mean values of sides are available :—

AB = 27921·1′ ; AC = 42350·1′ ; AD = 54662·9′.

CHAPTER II.

TACHEOMETRIC PLANE-TABLING.

22. The Tacheometer or Telometer is generally understood to be a distance measuring theodolite, that is, a theodolite fitted with stadia wires on the diaphragm. A great deal of work has been done by the Tacheometer, but it has this decided drawback, that the bearings or directions of the lines measured have all to be noted most carefully in a field-book and this has to be subsequently plotted, meaning extra labour, a double chance of mistakes creeping in, and no direct check in the field of the work as it progresses, which is possible with the plane-table. The Tacheometric plane-table with its sight rule head may be described as a graphic Tacheometer where the table is the lower plate, the sight rule the upper plate, and the sight rule head the upper works with vertical arc telescope, etc.

The stadia wires are usually engraved on the diaphragm so that the intercept on a staff viewed say 100 feet away will be 1 foot exactly. This is not quite true, except with the new internal focussing telescopes or with the addition of the anallatic lens, as there exists a small focal constant which will be treated of later. The advantages of this direct method of stadia measures are as follows:—It is quick and accurate. No destruction of property is entailed by dragging a chain through standing crops, gardens, etc. Measurements over the head of a crowd in a city street can be made by reading a 15-foot staff held high up. Mistakes in measures will be those of the surveyor himself. Nothing is left to outside agency, except the holding of the staff in required positions.

A description of the instrument is as follows:—

Parts of the Sight rule head.

23. C. Standard or central pillar of instrument (*see Fig.* 6).

cc. Two clamp screws to clamp the sight rule head to the sight rule.

J. Lower cross level.

d. Screw for adjusting lower cross level.

b. Slow-motion screw for vernier scale and vernier arc level (H).

a. Slow-motion screw for telescope.

J. Vertical arc level.
e. Small capstan head antagonistic screws for adjustment of vertical arc level.
F. Vernier arc.
E. Vertical limb or primary scale.
G. Upper level fixed on telescope.
f. Adjustment for parallax.
g. Position of diaphragm.
h. Telescope focussing screw.
i. Eye-piece.
k. Object end of telescope.
l. Screw to adjust upper level which works on an inclined plane.
m. Eye-piece to read vertical arc.
n. Hinged brackets.
p. Small knob to pull out slide of sight rule
A. Sight rule.
B. Parallel slide.
D. Telescope.

24. **Adjustments.**—Set up the table and level it by means of its legs and footscrews with the ordinary type of carpenter's level supplied separately in the box. The table will now be nearly level.

Adjustments for horizontal collimation.—This adjustment is to set the line of sight so that when the instrument is turned, end for end, or has its "face" changed, the line of sight will be in the same plane or that the vertical wire of the diaphragm will intersect the same object on both faces with the same reading as with a theodolite (the sight rule A with its slide B takes the place of the horizontal limb of the theodolite).

Set the sight rule over two footscrews and correct the cross level J by the screw *d* and having obtained a fair level for the table and instrument, bring into view some distant object like a rod or pole and clamp the axis of the table with the slow-motion screw beneath the table intersect the object on the vertical wire of the diaphragm and draw very carefully a straight line along the fiducial edge of the sight rule. The instrument in the first position used will be probably on face left (the usual working position of instrument) or, in other words, the vertical arc will be on the left of the telescope. Now turn the instrument end for end and transit telescope and set the sight rule edge again along the straight line. The instrument is now on the right face, and if there is no collimation error the vertical wire should intersect

the object selected, if not correct half the error by the slow-motion screw beneath the table and half by the diaphragm screws.

It is not necessary to obtain exact horizontal collimation as with a theodolite ; indeed, it is not possible, as what is appreciable in the telescope is not appreciable on the plane-table since the least error in the drawing of and setting on the straight line, will make a decided difference in the telescope. Care should be taken during this adjustment to note whether the axis of the instrument is vertical at the time of sighting object, that is, that the cross level (J) is in the centre of its run.

25. **Adjustment for vertical collimation tacheometric plane-table head.**—The methods of adjustment given for Theodolites have been considered complicated and tedious especially when through carelessness or ignorance both adjustments for levels H and G have been disturbed. It is proposed here to first deal with the adjusting of the instrument when it has been placed on the board which has been carefully levelled by means of the separate ordinary level supplied in the box. After this levelment the footscrews should not be touched and the sight rule head should be placed centrally on the table so that most of its weight is distributed over the footscrews.

Align the telescope on to some defined distant object and bring bubble J to the centre and also bubble H, intersect the object by using tangent screw "*a*" and record the reading. Next change the face of the instrument by rotating the telescope through 180° and changing the whole instrument end for end, bring bubble H again into the centre by tangent screw "*b*" and intersect object by means of tangent screw "*a*" read and record the reading.

Set the vernier to the mean reading by means of screw "*b*" and since this screw b, *and this is important to remember in this instrument*, controls the bubble H and not the line of sight, therefore any deviation of the bubble H will be corrected by the small adjusting screws *ee*.

Next make the vertical circle read 0° by means of tangent screw "*a*" and any deviation of the bubble G is corrected by its own bubble nuts when the line of sight will be horizontal. It will be noticed that the diaphragm has not been touched for reasons already given in Chapter III., Part I. of this work.

When the sight rule head is to be used as a level and thus adjusted it is necessary to release the screws "*c c*" and place it on the three-winged bracket which supports the table. Having done this it will be found that two wings of this bracket are unoccupied on which the separate bubble supplied, and already alluded to, can be placed and

the bracket thus approximately levelled first over two footscrews and then over the third when the footscrews should not again be interfered with and the adjustment carried out as given in detail above.

If the bubble J has also been interfered with it should be adjusted over one footscrew and then end for end and have the error corrected by its own screws. If the separate bubble supplied in the box is also out of adjustment, it can be corrected quite easily on an ordinary office table by noting the position of the bubble in one direction and then in an endwise direction and half the error corrected by its own bubble nuts.

When the instrument is used as a level and after it has been adjusted it will be found that the line of sight will be horizontal when bubble G is brought to the centre by tangent screw "*a*."

Next with the telescope over the line of the footscrew selected but with the eye end of the telescope towards the operator, note the position of the bubble G, and if the bubble is in the centre of its run the axis of the bubble is truly horizontal; if not, correct half the difference by the tangent screw *a* and half by the footscrew and repeat till perfect, that is, till the bubble remains in the centre of its run, no matter in what position the telescope points. If the tangent screw *a* has been manipulated to correct bubble, the 0° will of course, no longer coincide, and the error will be one of the vernier which, as will be seen, will be adjusted later.

(2). With the bubble G in the centre of its run read to any mark on a wall or for preference a level staff; next reverse or invert the telescope and turn the instrument through 180° (technically known as changing face) and bring the bubble G now lying underneath the telescope (new pattern instrument) to the centre of its run by tangent screw *a* and again read to the mark or staff. If the horizontal wire reintersects the mark, or gives the same reading on the staff the line of sight is horizontal, if not, half the error is corrected by the diaphragm, or for preference, the horizontal wire is set to the mean by the tangent screw *a* in which case the bubble G will no longer be in the centre and is to be corrected by the capstan nuts attached to the bubble.

The 0° of the graduated limb will now be in a true position with reference to the line of sight and the bubble axis which have been made parallel to each other and are thus both truly horizontal.

(3). With the tangent screw *b* bring the 0° of the vernier to coincide with the 0° of the limb, and if the vertical axis bubble H is not in the centre, unclamp the screw placed midway between the screws *ee* (in *Fig.* 7) and adjust the bubble H by the antagonistic screws *ee* of the

vertical arc ; adjust and reclamp the screw just mentioned which makes this adjustment permanent. In the latest pattern instrument the adjusting screws *ee* have been replaced by a single screw, but the method of adjustment is the same as described.

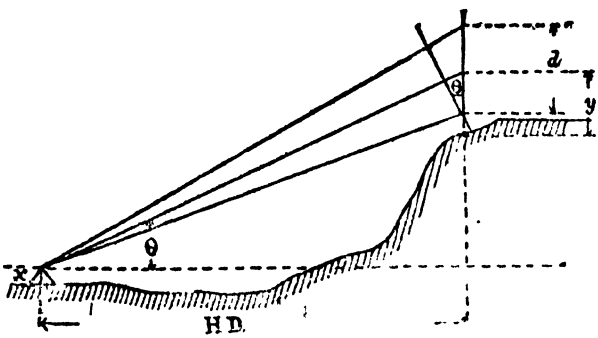
The instrument is now in perfect adjustment for vertical collimation so that when the bubble G is in the centre of its run the line of sight is horizontal, and when the vertical arc bubble H is in the centre of its run the readings to objects above or below the line of sight will be true elevations or depressions. It should be noted that any manipulation of the tangent screw *b* does not affect the line of sight in the telescope which is very convenient for work as when an intersection is made the bubble H can be independently brought to the centre, and the reading taking without any fear of the line of sight altering. The small bubble J at the foot of the standard should be made central by the screw *d* during the operations above described although any small deviation will not affect the adjustment.

26. **The Stadia.***—If D is the distance away of the staff, *d* the distance subtended on the staff and *k* the constant of the instrument stadia wires then

$$D = \frac{d}{k} = \frac{d}{\frac{1}{100}} = d \times 100.$$

Example—

Let the readings on the staff for the two wires be 5·28 and 3·47, then d = 1·81 and D = 1·81 × 100 = 181 feet. This holds good only if the staff and instrument are more or less on the same level, but if the vertical angle of the telescope is greater than 3° (there is no appreciable difference up to 3°) the quantity D will have to be reduced to horizontal distance, or HD, unless the staff could be tilted to be at right angles to the line of sight, but as this is impossible to any degree of accuracy, the staff is invariably held vertical and a correction applied as follows :— *Fig.* 9.



* *For* theory of lenses, etc., *see* Appendix I

If the telescope is inclined and the staff is held vertical the number of divisions intercepted would be more than if the staff was held at right angles to the line of sight, and the angle through which the intercepted portion of the staff would have to move from the vertical would be equal to the angle of elevation or depression of the telescope. Call this angle θ (*see Fig.* 9). The number of divisions read on the staff would have to be reduced by multiplying them by cos θ, the distance thus obtained must be again multiplied by cos θ, in order to obtain the horizontal distance.

$$\text{Now} \quad D = d \cos\theta \times k$$

$$\text{but} \quad \frac{HD}{D} = \cos\theta$$

$$\text{or} \quad D = \frac{HD}{\cos\theta}$$

$$\therefore HD = d\cos^2\theta \times k.$$

27. **The Stadia focal constant.**—Let AB represent the stadia wires on the diaphragm, and if lines be drawn from A and B through the optical centre O of the object glass, such lines will cut the staff at B′ A′. These lines are secondary axes If i represent the distance AB of the stadia wires and s represent the interval B′ A′ on the staff; then by similar triangles, $i : s :: f : d$ where $f =$ the distance from the diaphragm to the optical centre of the lens, and $d =$ the distance from the optical centre of the object lens to the staff A′ B′.

By the law of lenses $\frac{1}{f} + \frac{1}{d} = \frac{1}{F}$ (F being the principal focal distance).

From the above two equations we obtain $d = \frac{F}{i}\, s + F$. Now since D the distance from the staff to the axis of the instrument is required, the distance from the axis to the optical centre of the object glass or c must be added and since $D = d + c \therefore D = \frac{F}{i}\, s + (F + c)$.

Now if the wires in the instrument are fixed or engraved on glass the ratio $\frac{F}{i}$ usually $\frac{100}{1}$ is a constant, and if A′ B′ = 3 feet then the distance from the principal focus of the lens to the staff s will be 300 feet, and the distance D from the centre of the instrument to the staff will be 300 + (F + c). (F + c) may be taken to be 1 foot as it usually varies from ·75 to 1·25 feet, and since (F + c) is inappreciable on all ordinary scales of survey it need not be considered. It is for this reason that the anallatic lens * has been omitted by the makers as the addition of such cuts out a certain amount of light which is not counterbalanced by any real gain in utility.

* This has been overcome in the new India Pattern.

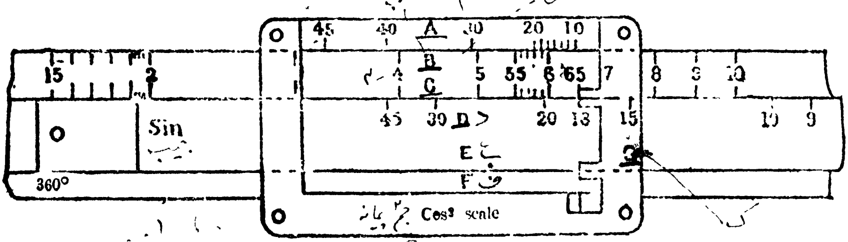
28. To obtain the difference of levels between ground level at staff and the ground level of instrument, let y = distance on staff from line of sight to ground level of staff and x equal height of instrument (*Fig.* 6). then difference of levels = HD $\tan\theta + x - y$. The angle θ will always be the angle of elevation or depression of the mean reading of the staff or the angle of elevation or depression of the central stadia wire.

Example—Let the readings on the two stadia wires be 5·28 and 3·47 (mean reading is therefore 4·38 and d = 1·81), and the angle of elevation or depression be 10°, height of instrument to telescope 5 feet—

$$\text{then HD} = \cos^2 10^\circ \times 1{\cdot}81 \times 100$$
$$= 175{\cdot}5 \text{ feet}$$
$$\text{and difference of level} = \text{HD} \tan 10^\circ + x - y$$
$$= (175{\cdot}5 \times {\cdot}176) + 5{\cdot}0 - 4{\cdot}38$$
$$= 30{\cdot}71 \text{ feet.}$$

This reduction to the horizontal distance and the method of obtaining difference of levels seems to bring in a lot of computation and rob the Tacheometer of its title as a time-saving instrument, but these reductions are easily obtained by two movements on a *slide rule* invented and manufactured by Kern et Cie of Aarau, Switzerland, as follows :—

Fig. 10.



29. **Slide Rule**.—To find the reduced horizontal distance place the index line of the traverser G on the scale C to the distance observed × 100 and read Scale B opposite the angle of elevation or depression of Scale A. In the above example place index to 181 of Scale C and read Scale B opposite 10° of Scale A = 175·5' (*see Fig.* 10) ; to find the difference in height between the telescope and the central wire (that is without taking into consideration height of telescope above the ground and the height of the mean reading above ground) set the asterisk of Scale D to the reduced horizontal distance, that is 175·5 of example on Scale C, and the index of the traverser G to 10° on Scale E, the traverser in this position will have its index on 30·9 of Scale C which is required to be known.

The larger pattern stadia slide rule made by Kern is about 20 inches in length and is more accurate than the ordinary pattern. The upper scale of the rule is for 360° and the lower scale for 400 grades. The slide has engraved on it the number which does for both the English and Swiss systems of measure. In the centre of the upper scale is found 0, to the left of 0 up to 45° is the $\cos^2$ scale, and to the right of the 0 is the sin cos scale up to 7° which is continued from the left of the rule to 45° of the $\cos^2$ scale.

One setting of the slide completes the operation. If the stadia reads 2 feet on the staff = 200 feet direct distance, set two of the slide at 0 of the upper scale. If now 20° was the vertical angle the horizontal distance will appear on the slide under 20° of the $\cos^2$ scale = 176·6, and the vertical height will be found under 20° of the sin cos scale = 64·25.

If heights are to be deduced from fixed objects it must be remembered that the distance of the object from the observer is taken off the plan and is thus not a direct distance or the hypotenuse, but the base of the triangle, and a tangent scale would be necessary in place of a sin cos scale. For all practical purposes the above slide rule is accurate enough and the following would be the procedure:—Let the distance on plan be 200 feet and the angle 20°. Set 2 of the slide opposite 20° of the $\cos^2$ scale and read the difference in height opposite sin cos 20° = 72·8′ opposite 0° of the upper scale will be given the direct distance or hypotenuse.

30. **Methods of Survey.**—The following examples of survey, Plates III. to VII. will serve to illustrate the use of the Tacheometric plane-table:—

Plate III. is a plan of a piece of country very common to India at an elevation of 1,000 feet above M. S. L. It might well represent the watershed portion of a catchment area for a tank irrigation scheme for which a preliminary survey on 1,000 feet to the inch scale is required with contours at 10 feet interval, the Reservoir site being perhaps 10 to 20 miles distant down stream. To arrive at any degree of accuracy over a large area of country the following procedure should be adapted:—

Having made a rough reconnaissance of the ground, D is found to be visible from C and E, and an extension on the base DE or CE in the direction of F is found to be practicable. A system of plane-table triangulation would meet with the requirements of the case, and from a selected base, extensions should be made in every direction to cover

the catchment area "*pari passu*" with the detail work. Let D, C and E be three plane-table triangulation stations and let them be flagged and marked. On examining the ground between D and C two positions A and B are selected from which D and C can be seen. AB can now be made the base line.

Set up the table at A, level it, and rotate the board to suit the convenience and direction of the work, place the magnetic compass on the board and at or near one edge of the board, pencil in the magnetic N and clamp table. Select a suitable position on the paper to represent A and mark the position directly beneath it "*in situ.*" Let a staff man be sent to B, sight B through telescope and intersect the staff on the vertical wire. Draw a line or "ray" AB through the point A so that the ray passing through A is parallel to the edge of the sight rule* and produce the ray in both directions, or leave short guide rays (shown in Plate II., as "ray BA," "ray AB," "ray DA," etc.)

These guide rays are very necessary at the commencement of any work where expansion from a short base is made as they confine the azimuthal error. Through the point A, rays and guide rays should be drawn to the positions D and C, and any conspicuous object on surrounding hillocks or on spurs, etc. Rays should be drawn, first the important ones, before any attempt is made at measuring the base, or taking angles of elevation or depression, because the slightest movement in azimuth or direction is certain to give endless trouble later; in fact, it is a good principle to complete the drawing of rays by returning to the original setting and checking it. The vertical readings should next be taken and neatly recorded along the ray drawn to the station to which the height has been taken: then follows the measurement of the base line.

The base line AB requires careful measuring, and this is best done by dividing up the line into two or three portions according to the length of the line. In the example AB may be conveniently divided up into three parts *a*A, *a*B, and *b*B with an average interval of 500 feet or so at which distance the staff can be clearly read to the second decimal place. With this in view a staff man is sent in the direction of B and is halted at "*a*" distance of 500 or 600 feet away, and readings to the staff for subtense distance, and a vertical angle to the reading on the staff equalling the height of the axis of the telescope, are made. These are recorded and the table moved to position *b* as

*To ensure correct rays, a very important precaution, keep the pencil at the same angle throughout the process of drawing the line as when drawing through the point A.

near as possible between A and B. Staff "*a*" and a staff on B are read in the same manner as at A, and the three distances are reduced to the horizontal distance which is plotted along the ray AB when the position of AB is found.

The table is now moved to position B *in situ* and first roughly placed in azimuth, the plotted point B is plumbed over the mark at B, by unclamping the screw from which the plummet hangs, and shifting the board laterally; the board is then levelled and the fiducial edge of the sight rule is placed exactly parallel to the ray and guide rays AB and BA previously drawn from A, the point A is intersected and the board clamped. The board is now in azimuth with reference to the initial azimuth taken at A. Sight to D and C and let rays and guide rays to D and C be drawn. The intersections of the rays from A and B to D and C will denote the position of D and C; the careful plane-tabler though, will visit D and C and also check them "*inter se.*" The ray work is completed at B when all the previously-sighted conspicuous points or such as can be seen from B, are intersected, and a return is made to the sight on A to make sure of the board having kept its azimuth. Vertical angles to D, C and A are read and the table moved to position D or C. Let position D be next visited. The table is put through all the previously-explained movements, and the azimuth here is not only checked by the guide ray from A but also the guide ray from B. If these guide rays agree exactly with the intersection of the respective stations A and B position D may be accepted as correct, and if the rays to C have also been carefully drawn C will intersect from D. Draw a guide ray DC and take rays to E and F, intersect all conspicuous objects and take heights, and the work at D is completed. At station C the table is put into azimuth by setting on the guide ray DC, and A and B having been checked and found correct, rays and heights to E and F and conspicuous points are again taken. A short visit to E is now paid so as to obtain a third ray to F and better cross intersections to intersected points. For example, the position of the tree lying within the triangle DEF is a little ambiguous till the cross ray from E finally settles its position. The preliminary work so far is now completed, and the guide rays may be rubbed out, the heights of stations calculated, and detail survey commenced. The method of deducing heights is explained in the reference made to the slide rule* which gives the difference of height between the axis of the telescope

* If the H.D is beyond the limits of the slide rule the heights should be calculated by taking off the H.D. from the plot and multiplying the same by the natural tangent of the angle and the correction for curvature applied (curvature = $\frac{2}{3}' d^2$ where d = distance in miles).

and the point observed on, or the object intersected by the central wire; and if the value is denoted by z and the heights of axis of telescope and point observed to above ground level by x and y respectively, then difference of heights of ground level $= \pm z + (x - y)$ ($\pm$ according as the station observed to is above or below B, that is, an Elevation or Depression, from station over which the table is fixed). It is usual to start the detail work from the high ground and work down to the streams, and with this in view the table should be taken to D as a starting point and a staff sent to I. (positions of table are shown in Plate in Roman numerals and positions of staff in small figures), the table at D is orientated or put into azimuth, and a direction and reading to the staff at I. is made, and thus the position of I. with its reduced level is known and plotted.

The table is now moved to I. and placed over the mark, no exact plumbing of point over mark is necessary, as the orientating of board on D is not necessary since E or F or some distant point is sure to be visible, and an inch or so error over the position at I. will make no difference in the azimuth setting. Place sight rule on the plotted point I., and the point on the board appertaining to the most distant point seen and intersect distant object.

Detail can now be put in by reading staves held at heads, bends and junctions of streams, bends of roads, etc., within reliable distance of the table. The distance as subtended of each staff is calculated and reduced on the slide rule and is plotted along the fiducial edge of the sight rule; the reduced level is then entered and detail and contours drawn in. No pencilling of rays is necessary, and with the new system of a sliding parallel ruler fitted to the sight rule, the telescope may be observed from within an inch or so of the station point, and the parallel slide then brought to the station point and the staff position plotted with a divider, thus accomplishing a great saving of time and at the same time conducing to cleanliness.

From station I., let positions 1, 2, 3, 4, 5 and 6 be fixed and a rough delineation of the 1,000 contour be drawn in by sending a staff man back towards D—for example, if the height to the axis of the telescope is 5 feet and the reduced level of I. is 1,002 feet, then when the horizontal wire cuts the staff at 7 feet approximately the staff distance subtended by the stadia wires is read and thus a point on the 1,000-foot contour is found—and again by reading the height of the instrument on the staff (5 feet) at 6, 5, 4 and 2 and noting the angle, the slide rule will give the difference in level when the 990 contour is found; or the 990 contour may be fixed from station II. Station II. is found

in the same way as station I, and the board orientated on the most distant point and the staff read at 7, 8, 9, 10, 11 and 12. Stations I. and II. have been fixed by ray and distance, and the magnetic compass so far has not been used. Suppose now at position 12 it was the intention of the plane-tabler to set up his board and make it his station III., but on arriving at the position he finds his view blocked in most directions so that he cannot see one distant point to set by, but on going to a position shown as station III. he finds that a distant point is seen and that also 12 can be seen. He sets up his table and orientates his board with the magnetic compass, this time reads the staff at 12 and plots his point or station III., and having found III. he adjusts his azimuth on the distant point if the compass variation has altered, (but any small alteration in the compass will not affect his short distance direction 12 to III.), so that this method of using the magnetic compass, when in a difficulty of this sort, is not inaccurate; but a whole series of points or stations fixed by magnetic compass is not accurate, and is only to be resorted to when the jungle is dense and the traverse so plotted can finally be closed on a reliable fixing and checked. In such a contingency, if the traverse point and the fixing made by resection do not agree, and if the compass variation has been constant, the starting point and closing point of the traverse and all detail fixed by it, should be traced off on tracing paper and the adjustment made on the starting point and true closing point, and the adjusted detail pricked or transferred through.

In Plate III. we will suppose that many alternate stations have been fixed in this manner only because it was unavoidable but that wherever the plane-table was set up a check on direction was made and at times a check on distance was also made by observing to C, E or F. The cross ray from these stations should pass through the plotted point of station; if it does not, a desirable spot should be chosen close by where at least three points can be seen and a check fixing made within the triangle by resection. The plane-tabler having arrived at station IX. finds, if he continues down the stream, he will be without the triangle of any three of his points, so he leaves the stream here till he has triangulated points further in advance and turns his attention to the other branch of the stream. He does not traverse back from IX. which would be a waste of time, but goes back to a suitable station or a staff position say No. 29, places a staff man and looks about for a good position where he can see a distant point and at least one other, if not two. Let X. be his next position of station where he puts up his table. He orientates his board with the magnetic compass and reads

the staff on No. 29 and plots his position, and the point thus found he sets on a distant point in case there is any compass deflection, and corrects his position if necessary by resection from one or more fixed points.

If he had not read to No. 29 but merely orientated his board on a magnetic direction, the chances are he would have had to solve a large triangle (*see* Chapter VI., Part I.) which is, except in the hands of an expert, sometimes a lengthy undertaking.

There is yet another method of setting the board and traversing, and is known as the "back and forward" ray system when the forward station is set on to the back by guide rays, but this leads to error if the lines are short and the station point is not exactly plumbed over the mark. It is used on larger scales and when distant points are not visible. The stations by this method must also be distinctly intervisible and a great deal depends on the sight rule, the collimation of which should be perfect, that is to say, if a ray is drawn to a point and the sight rule turned end for end and laid on the ray the same object should again be intersected when the telescope is rotated vertically.

The plane-tabler continues his work towards A and B eventually closing on A, and he has now filled in the detail more or less between BAD and the junction of the streams.

From A he can proceed in the direction E putting in the remaining unsurveyed portion of the main stream closing on E, and then returning directly towards C and surveying the small feeders and putting in the watershed, and when completed turn his attention to the other branch and work from station V. and close on D, etc. In surveys of this description it is presumed that there is a staff man working on each side of the operator, that is to say, there are at least two men on this class of work, and probably three. To work with more than three, would be a waste of time and might lead to confusion in the staff men's duties. To work with two men would be sufficient for the beginner. As a comparison in time—with the tacheometric plane-table 4 or 5 points with reduced heights would be obtained as against one chained distance with the chain and plane-table. If the work is inaccurate with the tacheometric plane-table the operator has only himself to blame, whereas chaining done by illiterate khalassies is a fruitful source of error which means loss of time and temper. On the one hand, the true horizontal distance and a reduced height is found by a slide rule, on the other a direct distance along uneven ground is found when the plane-tabler makes a rough calculation and reduction to true horizontal distance. This can

culation cannot be anything but rough as no correct allowance can be made for the sag of the chain under such conditions. There is also the distinct advantage with the new method of reading and giving a distance to a point on a contour. The surveyor who has had to follow a chain laboriously dragged from point to point along the edge of a steep piece of ground very often through dense intervening undergrowth or from spur to spur, down and up steep inclines and over rugged ground, will appreciate an instrument that will give reduced and true distance across obstacles of this nature and the staff man when proceeding to the next point can halt at detail and have his staff read. Again, the staff need not be on the ground but can, if hidden by undergrowth, be held up vertically above a man's head, though a Sopwith's telescopic 15 feet staff should be sufficient to defeat all such contingencies. The best plane-tablers, except when working on very small scales, resort to the chain as a quick method of picking up intervening detail, and also as a quick aid to fixing and more often, if a fixing by resection is not possible, in the hope that his chained distance will supply the requisite data till such time as he finds a suitable spot to fix in and close his traverse. A distance measurer fitted to a sight rule which has a telescope fixed to it and a graduated staff suited to the scale of survey, must be immeasurably superior to the ordinary sight rule and chain, and must be a distinct advance on the older and less accurate method. Plate III. thus illustrates a system of plane-table triangulation and detail work on a scale. which is perhaps small to the Engineer, and which scarcely brings out all the advantages of the Tacheometric plane-table as contours at 10 feet interval require only a rough levelling. It is when the Engineer has to survey on a large scale and do contouring at 1 foot interval, say of his Reservoir basin, in order to find the cubic capacity of water impounded by a certain height of weir that the tacheometric plane-table shows up in its true light as a valuable survey instrument.

31. **Plate IV.** is a piece of riverain work on a scale of 500 feet to the inch with contours at 1 foot intervals. The plane-table stations are in Roman numerals and staff stations in small numerals, which, when followed consecutively, will show how and from which stations the detail has been surveyed. Heights of stations are reduced levels to the second place of decimals, so also some of the heights near the water level; the heights to the remaining points need be only to the first place of decimals to delineate the contour and need not be necessarily inked in unless the rises and falls are very marked. This plate

would also serve to illustrate a survey of a basin area if the high bunds or embankments were imagined to be the line of separation between the flat and high ground.

Let the table be set over bench-mark 990·86 and a point on the paper selected and marked I. to represent the position of the B.M. *in situ.* The table should be levelled and the axis of the telescope from the B.M. measured and recorded to the second place of decimals (the height of the telescope to the underneath portion of the table is always a constant quantity and it is only necessary to measure the height from the station *in situ* to the table). The board is now set to the given azimuth if any or to Magnetic N. and the staff on station II. is read for subtended distance and direction only and plotted and accurate guide rays are drawn. A height is not read to II. as the difference of level is greater than the length of an ordinary levelling staff (*see* Plate), and a reduced level by the slide rule is probably not accurate enough to two places of decimals so for this station recourse is made to an intermediate staff being read to, just as in ordinary levelling, and the plane-table again set up in a convenient position so that the difference of level is read on the staves.

If necessary station II. could be read direct for reduced level, and the result compared with the levelling method employed to make sure that no mistake has been made, but it can only serve as a check and not as a value from which a mean could be deduced. Having found the reduced level of II. the plane-table is placed over station II. and roughly turned in azimuth so that the plotted point II. can be plumbed accurately over the mark and levelled. If this rough orientation had not been made and the plotted point first plumbed, then when the board is put into azimuth the point will no longer be over the station mark. No distant point being previously fixed, the survey must be carried on by the "back and forward ray" traverse system, in preference to magnetic compass setting, which is, as has been explained, accurate if the distance between stations are not too short for the scale, and if the plumbing of the board is carefully done, stations intervisible, and there is no collimation error in the sight rule. The collimation error in this sight rule can be reduced to a minimum by the method already given. The sight rule is placed along the guide rays I. to II. and station I. is accurately intersected and the table clamped.

A staff man is sent to position 1 on small island and is told to leave an impress of his staff there. A reduced level to two places of decimals is made and the position of 1 plotted. The reason for this is that this

reduced level and position will act as a closing check from say station XII.

The detail is surveyed in the manner already explained for Plate III. using here the back and forward ray with guide rays for setting, and the work is checked at station XII. on position 1 and continued to XIII., and probably again checked and closed on position 10. To obtain the same plan with reduced levels a level would have been necessary, and with the ordinary plane-table and sight rule accurate chaining across water would have been impossible. The levelling would take an extra day to do, and it would then have to be subsequently plotted. The numerous points fixed by the tacheometric plane-table would, under the old system, require flags placed so that they could be intersected from another station or two or more to be really accurate, and the chances are a second view might not have been obtainable, and the position of flag abandoned without considering probable confusion of flags. The saving in time and therefore the saving in hired labour would soon repay the initial purchase outlay of the new instrument the cost of which, at present, is the only objection to it, but its apparent prohibitive cost is as nothing compared to the savings it must make before even it has lost its first polish and shows signs of a little wear and tear.

32. **Plate V.** is part of a plan of a Revenue or a Cadastral Survey showing fields.

T_1 T_2 T_3 T_4 T_5 and T_6 are theodolite traverse stations along the outer boundary of the village, which boundaries are invariably traversed by a theodolite, and a record kept for future reference, so that if marks are removed and disputes arise as to the ownership of land, the boundary can be laid down from the field-book.

The subsequent surveying of the field boundaries need not be again explained, as the reader now will be able to understand how the different points have been fixed and the accuracy of the survey checked. The Roman figures represent the positions of the table and the small figures the positions of the staves.

33. **Plate VI.**—The triangles and lines in red show the system of triangulation carried along the river Rámganga of the Sardah-Ganges feeder scheme. The triangulation in this case would have been plotted by rectangular co-ordinates as the series was a fairly long one, but if the plate, as given, had been the full extent of the survey, it would have been necessary only to have plane-tabled those positions as explained for Plate III. A system of levelling has also been run from station to station, and the leveller has left here and there pegs on his route to be an

additional check to the plane-table heights. These are shown on the plate to the second figure of decimals.

Let it be supposed that detail work up to the line B C has been already completed, and that the next piece of country to be taken up conveniently from the camp is the stretch of river lying between the points B, C and D. The table is put up at B and for purposes of the scale need only be roughly centered at B. Since B is a triangulated station the points B, C and A are visible. Select the most distant point and let it be C. Place the sight rule edge along the line BC and unclamp the table, and twist it so that the point C is intersected in the telescope. The table is now in "azimuth" or in true orientation.

The surveyor is now ready to commence his work, and staff men are sent out, and the positions 642·6, 641·8, 640·5, 641·0, are fixed and the position of the low ridge running NNE is also found. He finds he has no more work to do at B, and as he might require his magnetic direction, he puts on his compass* and marks the edge of the compass box as his direction of magnetic north. As he wishes to travel in a north-easterly direction with the probability of not seeing more than one distant point, the staff man at 641·8 is not moved. We will suppose that a position (shown as I. in the Plate) is selected and that D can be seen, also A and B.

The surveyor can thus obtain the true position of 1 by resection or what is usually known in India as a "fixing." He is within the triangle BAD, so the true position will be within his triangle of error, if any. To solve this triangle by the usual method would be a waste of time under the conditions, and what he should do is to put on his compass and orientate his board till the needle is steady at magnetic north, when the point 641·8 is next sighted and read, and his position plotted from 641·8. The point thus found for 1 should be used with D to correct the azimuth of the board in case the magnetic compass gave a variation and the position of 1 can now be checked by resection from B and A. There should be no difference for even had the variation altered much it would scarcely affect the short line between the position 641·8 and 1.

At station I. the staff men are kept busy as there is a good deal of work to be done here. On completion of the work at I. the plane-tabler finds that before he can move his table northwards it is necessary for him to make another station to the east. On this occasion he keeps one staff man at 641·0 with the instructions to halt there till he is given

* The compass supplied by Kern is what the manufacturer calls a declination compass. The needle is six inches long, is very sensitive and beautifully balanced and should not be unclamped except on the table after it has been approximately levelled.

orders to move, and he uses the staff man at 640·4 to find his position 2 exactly as he did in the former case in finding his position 1.

Now at position 2 he probably sees the point A to check his azimuth, but in case he does not, he can accept his magnetic compass direction as he is not going to use 2 to continue his traverse, and any points he fixes from two will not be affected by a slight deviation in the compass. The surveyor now selects the level peg with the value 641·17 and finds his position from point 641·0 and checks it if possible. He reduces his height from 641·0 and checks his height and correct it if necessary by the height given by the leveller. He continues in this way and closes eventually on C, checking his position and heights wherever possible, and having an intermediate staff to obtain an approximate position by, and thus being always a convenient distance *beyond* the positions fixed from the last station.

34. If his table were not set up at alternate stations as might happens in dense jungle, when the back and forward ray system would have to be resorted to, only half the quantity of work would be possible. At C the surveyor returns eventually closing on his own work at station 4 making a digression to station II.

35. In the same way, a cantonment survey can be made on a theodolite traverse as a basis and corners of buildings and compounds, the position of lamps, drains, boundary marks, hydrants, etc., can be fixed on the plane-table. Standing crops and private gardens are not damaged as they would be with a chain dragged from point to point. The staff held above the level of the heads of people in a crowded thoroughfare would be all that was necessary to fix a street corner, and further a complete system of reduced levels would be forthcoming.

36. Stress has been laid on the desirability of keeping the board in correct azimuth because there is little or no error in the distance which is a horizontal one, and which is tacheometrically obtained, provided the staff is held within the distance of the lens power of the telescope, and too long shots are not taken to get over the ground quickly. In traversing with the theodolite and chain in fairly level country the angular accuracy is more or less balanced by the chain accuracy, but in hill work this balance does not hold good unless the chaining is done scientifically as on a base line. The distance the chain measures is in excess or defect (excess generally) of the true horizontal distance. The excess in one direction becomes an angular correction when the traverse assumes, say, a direction at right angles to the former one, and so the closing error of such a traverse is a fictitious

one. As most of such traverse work is, in India, carried on in forest-clad hills where triangulation is impossible except to serve as a tie to the theodolite traverse, and since forest fire lines exist the tacheometric plane-table should be adopted, each plane-tabler doing a recognised block bounded by fire lines and working on the basis of part of a theodolite traverse joining triangulated points if available on the ridges which are usually fairly flat. A good series of heights as a check to contouring would also be carried on *pari passu* with the traversing.

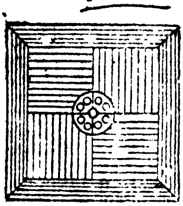
A theodolite traverse on the ridge is unnecessary if the block is not to be geographically fixed.

37. Only the bare outline of the possibilities of this system of plane-tabling has been touched upon, and the reader will find in Chapter VI, Part I., fuller details. Every plane-tabler learns by experience many short cuts, and it is left to him to discover them and perfect his knowledge in this class of work, and before closing this chapter it would be as well to give a few further hints on plane-tabling in general, and the difficulties that have to be met when working with imperfect sight rules and on plane-tables made of unseasoned wood.

Plane-tabling is usually based on triangulation or traversing, and the paper on which the survey is made should be pasted on cloth and the cloth on to the board and left to thoroughly dry before the computed points are plotted, care being taken that the *board* is not wetted. The ordinary plane-tables are made up of three pieces of pine with grain of wood in one direction braced underneath by two battens of teak running laterally with slot holes to allow for expansion and contraction, usually contraction, since during the rainy months and at night the wood absorbs moisture and becomes swollen, and this moisture is given off in the dry season and during the heat of the day. The uneven contraction in one direction, breath-wise generally, renders the best triangulation or traversing useless directly the board begins to give, and the only way to surmount this difficulty is to spend one to seven days, according to the scale and area of country to be surveyed, in visiting fixed stations and points in different portions of the board, and from them cutting numerous auxiliary points, so that no part of the board is left without points 2 to 3 inches apart. As long as the surveyor finds, during this preliminary work, that his distant points are correct, he can continue working with all and any trigonometrical data, but when he finds distant points disagreeing, then he must reject them and confine himself to his *near* points only. In this manner he breaks up his triangulation into smaller triangles, and when he feels that his board is "giving"

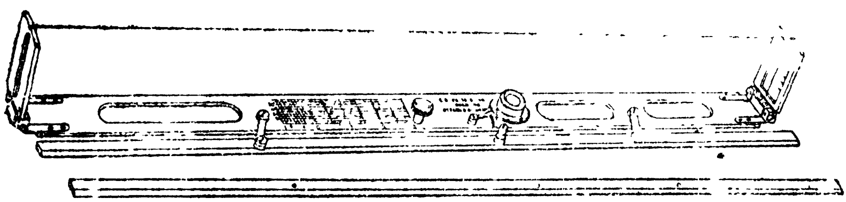
he must use only close points to work with and never attempt to set on or cut in a distant point. This compulsory hemming in of work robs plane-tabling of its greatest charm, and prevents the surveyor taking long shots to distant peaks, prominent temples, trees, rocks, junctions of streams, etc.; and what is worse the examination of his own work from a prominent point which is always a pleasing occupation.

A plane-table made of four pieces of good pine (Linden wood preferred) set into a frame of hard wood, the grains of the pine to run crosswise, *vide* sketch, is suggested as more or less an efficient and cheap way of getting over this difficulty, but nothing short of an aluminium table as nearly square as possible, will obviate the difficulty of slew in the plotted points.



If the sight rule is of the plain ordinary pattern the wood should be of the best straight-grained seasoned wood procurable—boxwood is as good as any and does not sweat and leave unsightly marks on the paper. The line of sight through the sight vanes should be parallel to the fiducial or working edge of the ruler, and if it is not so, the back and forward setting of a ray will not be the same. If a sight rule has warped, only a selected portion of its edge should be used for drawing rays and the work examined by the same rule.

The following design* is recommended as it can be adjusted for collimation, and the sight rule need not be held against the pencil or pin over the station point for drawing rays It is made of electrum



with a narrow parallel attachment jointed so that it can be adjusted to the line of sight. The parallel attachment can be removed if damaged or exchanged for an extra edge carried in the box. The sight vanes are strongly hinged so that they can be closed down, and both rules have a button attachment, one at the point of balance to lift the rule by and the other for the parallel slide, besides a small bubble to set the table into fair levelment.

* India pattern manufactured by E. R. Watts and Son, London. The College pattern is made of box wood and instead of the engraved scale a magnetic compass is fitted.

38. The plane-tabler should not be in a hurry to start his detail work. He should first visit as many fixed points as possible, check them and then spend a day or so, according to the scale, in fixing supplementary points. He should avoid working "without the triangle" and rather than do so, he should endeavour to fix a point outside his work so that he will be "within the triangle" in fixing. The solution of a fixing "without the triangle" is correct in theory, but the slightest warp in the table may give a grave error, and this cannot happen to a fixing "within the triangle" as either the triangle will not solve or the error is confined to a very small quantity. In most cases the fixing without the triangle is ambiguous and therefore should be avoided. For quick work in open or fairly open country the best system is, after obtaining a fixing from a known station, to start the chain men measuring on to any distant conspicuous object (which need not be a plane-table fixed object) in the direction in which the work lies and to take a ray to the object selected. The chain should be halted at detail, and a position along the chain selected where at least one distant fixed point is visible and probably one or two close ones. He should note the distance measured and plot along the ray. He now orients the board by any distant fixed point and intersects by a close point; the intersection will be a check on the measured distance. If more than one close point is visible and there is no reason why there should not be if the preliminary work has been properly done, the chained distance plot now becomes a reliable fixing. Directly the position is found, the chain men should be started on the next line and with a little experience, they will halt at likely detail and wait, the plane-tabler in the mean time busies himself with the detail near his board.

The chain is therefore only a means to an end, that is, quick-fixing, and the work is not dependent on chain measure except perhaps on an average of one or two stations in every ten, but even so the very next position will probably serve as a check. On small scales this system is advocated so long as the surveyor has one fairly distant point in view at each station which need not be the same point every time. Working up and down inclines on this system of accepting the chained distance and setting the plotted point thus found on a distant point for azimuth, the horizontal distance need not be calculated if a close-fixed point to the right or left of the line is forthcoming from which a ray is taken. The intersection of this ray with the previous one for direction will give the true point, that is, the chained distance will now be reduced to horizontal distance. The board is now corrected

for azimuth by the acceptance of the new or true point, and this slight alteration of the original setting will not affect the cross ray, but if it does the grinding-down process can be continued till no alteration takes place. It is better to put up your board and find a position by this method in the midst of your detail than to waste time, say 50 or 100 yards away, looking for a suitable spot to obtain a fixing by resection : such a check may be made later if there is any doubt about the accuracy of the chained point.

39. Pencil work should be of the finest and with the best hard graphite pencils ; the pencil point should be kept sharp by repeatedly rubbing it on a piece of sand-paper,* a strip of which can be glued to the legs of the table. When a pencil works soft, as it will do in hot dry weather, a new one should be cut and the old one put away for work in moist weather. Good soft rubber should be used which will not destory the surface of the paper, thus ensuring good ink work latter. A spare piece of rubber should always be carried. The pencil held upright on its base should be used to pivot the sight rule on when taking rays, and as the exposed lead will make ugly smudges on the paper, this should be wormed out with a penknife.

Avoid deep pin pricks and unsightly holes made with the dividers in plotting distances. Pencil work which is not final should not be inked in as erasures are not permissible. Eye-contouring will be done by taking heights at intervals with any of the ordinary instruments usually used, among which the survey pattern clinometer or tangent clinometer is recommended as accurate, but which cannot, of course, compete with the roughest levelling or the heights obtained by the Tacheometric instrument based on ordinary levelling and triangulation. The surveyor should thoroughly satisfy himself that he has completed the work at a fixing before leaving it, and as he walks from point to point he should take note of every twist and turn, may be in a stream, pathway, or road so as to be sure that nothing has escaped him or been omitted. He should observe the different aspects, that objects have from different views, and while he is at work his mind should be concentrated on it, and his aim should be extreme accuracy. He should train himself, when walking, to judge distances so that he can arrive at a good approximation of the distance away of an object on his map. Too much stress cannot be laid on eye-training. The faculty for judging distances, combined with good draftsmanship, is the essence of plane-tabling. Plane-tabling is an art not to be learnt in a day, and perfection is only reached after

* Winsor and Newton sell blocks of sand-paper which can be used for this purpose.

months of steady toil very often under the most trying conditions, but sufficient can be learnt in a very short time to answer all the ordinary purposes of an Engineer.

40. Surveys of projects and schemes likely to change the features of a large area of country should be based on Survey of India data which can be had from the head offices of that department. No department can be expected to accept work for incorporation in its maps if the basis of the survey is not above reproach, and the crudest survey on a large scale, when surveyed on a correct basis, may be of service when reduced to a small scale. A project or scheme should be based or connected at intervals on Survey of India data, and a gridiron or simple system of triangulation run by the Engineer as a basis for his plane-tabling work. This triangulation need only be computed to the value of sides and the sides plotted by arcs by means of beam compasses to obtain the third station. Every station should be levelled to, and level values left at intervals here and there between stations to act as a further check, if fairly exact reduced levels are required. Any datum value can be accepted for the starting stations of the levelling, and the difference recorded on each map when the levelling has been closed on either G. T. levelling or a bench-mark obtained from G. T. values. The height values given by Survey of India triangulation cannot be accepted as correct, and are only approximately so to within five feet, although the approximate value, as given, could be accepted for the initial station when the difference on closing on a B.M. would not be so great as to make the correction an unwieldy figure.

To enter more fully into the matter we may presume that an Engineer possesses the standard map or maps of the area over which he is to work, and also has a copy of the Auxiliary tables of the Survey of India which contains the information he must need for projecting and plotting his sheets. He subdivides his standard map into convenient squares or "graticules" of latitude and longitude to suit his scale, and projects on a piece of paper a graticule for each section of latitude (3′ 45″ or $\frac{1}{16}$th of a degree for 4″ scale, 1′ 52½″ or $\frac{1}{32}$nd of a degree for 8″ scale, and so forth). This piece of paper can be used to prick off any number of sheets for the same parallel of latitude. By subdividing this graticule and making scales he can plot the values of triangulation supplied him by the Survey of India. He proceeds now to break down his triangulation by a smaller system of his own, or if the country is open enough and he has further trigonometrical data to connect to, by a system of plane-table triangulation. The leveller may accompany him on this preli-

minary work so that he can get an idea where the station points are and where other intermediate stations are required.

This is the best and most scientific way of working, but in the event of trigonometrical points as fixed on the standard map, having too long a side *inter se* so that points as a rule would fall off the size of paper used for the scale, one point only could be taken and a base measured and the direction of the base fixed either from true or magnetic north. This would give the first two points of the survey one of which has a fixed value. The initial direction taken must not be again altered, since the position of points are in the future plotted by distances only, and since this is so the triangles should be as nearly as possible equilateral.

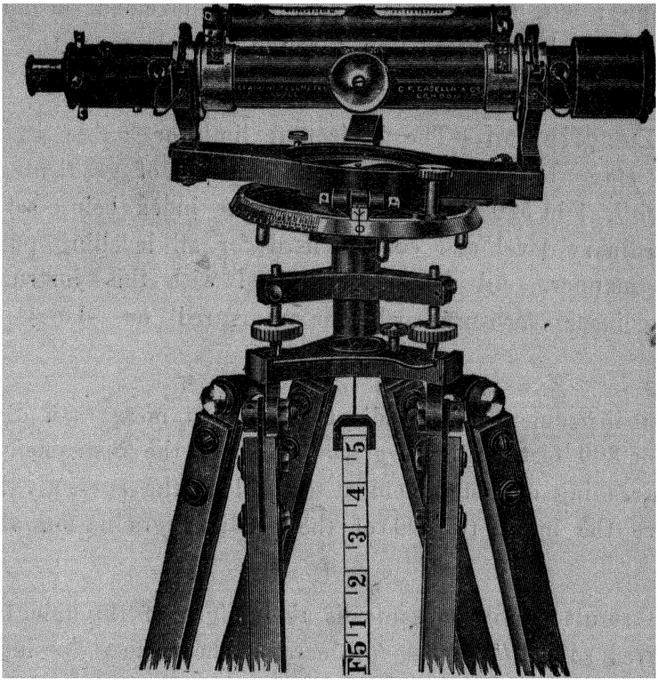
This triangulation should be connected where possible to any other trigonometrical data, or the plane-tabling based on such triangulation should endeavour to pick up such data. The survey work could be then utilised for correction of the standard map, and the engineer could see how he is progressing by plotting his work on a smaller scale, and fitting it on the trigonometrical data previously supplied.

With the hints given it should not be a difficult matter for the beginner to start work, and it must be remembered that progress at the beginning will be slow, but steady application and an awakening interest in all the possibilities of this art of surveying will make the long day short and work a pleasure. Such methods as chain and compass with field-books, etc., will seem as mere stepping stones to higher things, and the stadia method combined with topographical methods may be said at the present day at least, to be the last word in delineating ground.

41. **Plate VII**. is a piece of work done in moderately difficult ground to serve as a check on work by students, and the reader should notice that the triangulated points are those worked out in Chapter I. of this book. Plate VIII. is that of a staff suggested for this class of work.

42. The Gradient-Telemeter Level.—In this instrument,

Fig. 11.



made by C. F Casella and Co., London, as shown in the illustration above, are embodied, by an ingenious construction, all the means necessary for taking gradients, measuring or setting out distances, and obtaining differences in level, all of which are performed, in an extremely simple and rapid manner, by one and the same observation. With the Gradient-Telemeter Level the necessity for using a chain or tape is entirely removed, and as the operations are performed with singular accuracy, rapidity and ease, a much greater quantity of work can be done in a given time compared with the usual methods employed by surveyors and civil engineers.

The linear distances can be obtained far more accurately than with the chain, and this regardless of rough and broken ground, or the existence of a stream or other water, between the observer's station and the distant object.

A marked advantage with this instrument, and one that renders it singularly attractive when employed in the field, is, that there are *no calculations* to be made in connexion with its use; there is no *fine*

micrometer screw to work with its *certain* errors and elaborate calculations; there are no moving etched or spider lines which soon get broken, thus rendering the instrument useless until it can be returned to the maker to be set right: but, by a simple revolution or movement on the axis of the instrument and an observation through the telescope, the gradient, distance, or difference of level, is at once obtained.

43. **Description**.—The horizontal limb or circle immediately below the stage has engraved upon it a series of gradients, both of *rise* and *fall*, 1 in 500 to 1 in 24. On the index being set to zero it is an ordinary level and can be used for all levelling purposes in the usual manner. All distances obtained with this instrument are horizontal linear measures whether measured on slopes or level ground.

To put instrument into gradient adjustment measure a distance of 200 feet or 300 feet on level ground, and set the instrument to zero and take a reading on a staff held at the end of the correctly measured line. Move the index to 100 of the gradient graduation and again read staff.

If the result is not the same as the length of the line the index can be moved so as to increase or decrease the value. The index being set for this particular line should be correct for all readings of the instrument.

The following are the *gradient pairs*, which, when used, give by the difference of readings on the staff multiplied by 100 the horizontal distance away of the staff from the instrument :—

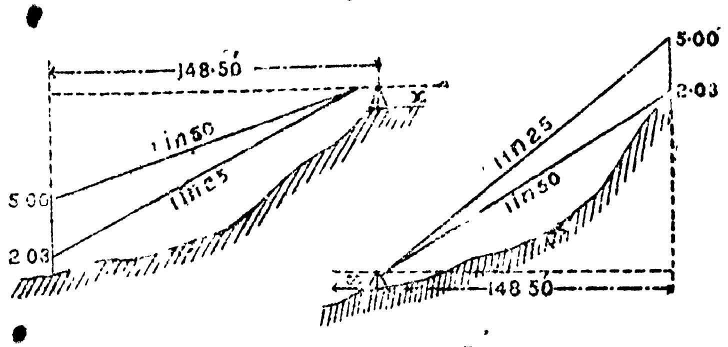
$$\left.\begin{matrix}100\\50\end{matrix}\right\} \quad \left.\begin{matrix}66\frac{2}{3}\\40\end{matrix}\right\} \quad \left.\begin{matrix}60\\37\frac{1}{2}\end{matrix}\right\} \quad \left.\begin{matrix}50\\33\frac{1}{3}\end{matrix}\right\} \quad \left.\begin{matrix}33\\25\end{matrix}\right\} \quad \left.\begin{matrix}25\\20\end{matrix}\right\} \quad \left.\begin{matrix}20\\16\frac{2}{3}\end{matrix}\right\} \quad \left.\begin{matrix}12\frac{1}{2}\\11\frac{1}{9}\end{matrix}\right\} \quad \left.\begin{matrix}11\frac{1}{9}\\10\end{matrix}\right\}$$

or again, if a gradient and its half is used, the distance intercepted on the staff multiplied by the figures of the higher gradient used gives the same result—that is, if $\left\{\begin{matrix}50\\25\end{matrix}\right\}$ gradients are used and the distance intercepted on the staff = 2·97 then $h\ l\ d = 50 \times 2{\cdot}97 = 148{\cdot}50$ feet.

To obtain the difference of height between the telescope and a particular reading of the staff intersected by the horizontal wire, it is required only to multiply the $h\ l\ d$ by the gradient (taking the gradient as a fraction).

To illustrate this let us consider two cases—one of a "rise" and the other of a "fall"—with the above example.

Case 1 (fall). *Fig.* 12. Case 2 (rise).



Case 1.—With gradient 50 let the reading on the staff be 5·00′ and with gradient 25 let the reading on the staff be 2·03′, then the *h l d* = 148·50′ and height of reading 5·00′ on the staff below telescope = 148·50 × $\frac{1}{50}$ = 2·97′; therefore ground level of staff below telescope = 2·97′ + 5·00 = 7·97′.

Again, height of reading 2·03′ = 148·50 × $\frac{1}{25}$ = 5·94′ below telescope: therefore ground level or foot of staff = 5·94′ + 2 03′ = 7·97′ below telescope which is the same result as obtained with gradient 50.

Case 2.— With gradient 25 height of the staff at 5·00′ = 148·50 × $\frac{1}{25}$ = 5·94′ above the axis of the telescope; therefore the height of the ground level of staff above axis of telescope = 5·94′ − 5·00′ = ·94′ and similarly with gradient 50.

From which we obtain the following rule:—if x = height of telescope* (axis) above ground level of station over which instrument is placed, y = reading on the staft and z = difference of height between axis of telescope and staff reading, then—

ground level at instrument + $x - y \pm z$ = new ground level of staff

$$\left(\pm z \text{ according to } \frac{\text{rise}}{\text{fall}}\right)$$

Example.—Let ground level of instrument or station point oer which instrument is set = 1,000′ and let x = 4·00′.

Then ground level at staff with gradient 25.

in Case 1 = 1,000′ + 4·00′ − 5·00′ − 2·97′ = 996·03′,

in Case 2 = 1,000′ + 4·00′ − 5·00′ + 5·94′ = 104 94′.

---

* This is easily obtained by means of the plummet with tape attachment supplied with the instrument.

CHAPTER III.

PRACTICAL ASTRONOMY.

Introduction—Spherical Trigonometry.

53. **Definitions.**—A *sphere* is a solid every point of which is equidistant from a certain point within it. This point is the centre.

A *diameter* is a line drawn through the centre of the sphere and terminated at its extremities by the surface; a *radius* is a line drawn from the centre to the surface.

Great circles are those whose planes pass through the centre of the sphere, all others are small circles. The diameters of great circles are diameters of the sphere, and the diameters of small circles are not diameters of the sphere.

A line passing through the centre of a sphere and perpendicular to the plane of a circle of a sphere is called the *axis* of that circle. This line is limited by the surface of the sphere, and its extremities are known as the *poles* of that circle.

The *distance of two points* on the surface of a sphere is that portion of the arc of the great circle passing through the two points intercepted between them. A *spherical angle* is the angle at a point on the surface of a sphere formed by arcs of great circles passing through the point and is measured by the inclination of the planes of the two circles.

A *spherical triangle* is a triangular figure formed on the spherical surface of a sphere by arcs of three great circles each of which is less than a semicircle. As its angles are angles contained between the planes or are solid angles the triangle may be imagined to be a solid triangle for purposes of demonstration. Again, as the sides of a spherical triangle are arcs of great circles of the same sphere their lengths are proportional to the number of degrees contained in them, and hence these sides are usually calculated by the number of degrees, minutes and seconds they contained or subtend at the centre and are expressed in angular measure.

54. **Formulæ.**—From the foregoing definitions it will be understood that Spherical trigonometry treats of the ratios between sides (arcs

of great circles) and angles of triangles situated in three or more planes inclined to one another and passing through one point, (the centre) and as plane trigonometry treats with ratios between angles and sides of a figure in one plane, it might be said that spherical trigonometry bears the same relation to plane trigonometry as solid does to plane geometry.

The spherical triangle to be dealt with in this chapter is the spherical triangle PZO, where P is the pole, Z the zenith and O the object (sun, moon, planet or star), and if the earth is considered as the point where the planes of the spherical triangle meet, these planes are imagined to cut the surface of a great sphere (the celestial sphere) in three arcs of great circles and lines joining these points to the centre contain what is known as the solid angle.

By definition, these arcs, by their intersection, will form the spherical triangle PZO the sides of which are expressed in angular measure and the angles are those contained between the planes.

To prove some of the formulæ useful for astronomical work as required by the engineer and topographical surveyor it is intended to simplify the theoretical part by the use of solid geometry by which the solid angles are reduced to plane angles.

Note.—It is necessary that the reader should understand that in solid geometry a certain system of lettering is used, and a dash to a letter denotes an elevation, and a numeral added below the letter, different positions of the letter in plan in one and the same plane. In the diagram Z is the plan, Z′ the elevation of the point Z or the point immediately above Z. Z_1 Z_2 Z_3 Z_4, etc., are different positions of Z on the horizontal plane, and a dash added to one of these is the same point in elevation, *e.g.*, Z'_5 is the elevation of the point directly above Z_5. Again, the reader will understand that since Z_1 Z_2 Z_3, etc., are all different positions of Z that when such points are rotated or brought into position with one another they will coincide and become Z′ or the elevation of Z. In the perspective view only Z′ is given which in the other diagram is also Z′ immediately above Z.

In *Fig.* 1 of Plate X, let PZ′ OE be the solid triangle the face POE lying on the horizontal plane (H.P.) at a level of the earth's centre of which the sides PO (z), OZ′ (p), PZ′ (o) are given in angular measure. The first thing to do is to develop this solid triangle or to lay it down flat into the H.P. With E as centre and any radius EP or EO draw an arc Z_1POZ_2 and set off the angles Z_1EP, PEO and Z_2EO as given. The sector EZ_1POZ_2 represents now the development of the solid triangle EPOZ′. Z_1 and Z_2, as mentioned above, are as seen when the position of Z′ is rotated down into the H.P.

Let Z_1 and Z_2 be again rotated up with PE and EO as hinge lines respectively and let us consider what happens. The points Z_1

and Z_2 will have their plans following a path Z_1AZ' and Z_2BZ respectively, at right angles to their hinge lines, that is, that Z_1AZ' is at right angles to PE and Z_2BZ' is at right angles to OE, and to reconstruct the figure again from the development given draw Z_1Z and Z_2Z' at right angles to PE and OE respectively, and the point of intersection Z' (which is the elevation of Z in plan) is the elevation of Z_1 and Z_2 when they occupy a position directly above Z. To understand this more fully cut along the lines PZ_1 and Z_1E and fold along PE, and in the same way cut along OZ_2 and Z_2E and fold along OE and flap these over so that Z_1 and Z_2 coincide; and as EZ_1 and EZ_2 are radii of the same circle, EZ_1 will coincide with EZ_2 and become EZ' or the *intersection line of the two planes.*

Consider next the paper model as a solid and pass a vertical plane through EZ' down to EZ. The view we would obtain by looking at it directly from the right would be the view drawn on the left hand side of the diagram, that is, we have a side elevation of the vertical plane. To construct such a view draw a line $X'Y'$ parallel to EZ and draw EE at right angles to $X'Y'$ and with E' as centre draw an arc equal in radius to EZ', EP or EO. This arc is an elevation of part of the sphere. Through Z' (or Z) draw $Z'ZZ'$ parallel to EE' and where it cuts the arc or at Z'' the point will be the elevation of Z' and will represent the height of Z' above Z. Join $E'Z''$ and this will represent the elevation of the intersection of the two planes $EZ'P$ and $EZ'O$.

We have now found the height of Z' above Z, that is, the height of Z'' above the line $X'Y'$.

One of the rules of solid geometry is that if a plane is passed at right angles to the intersection of two planes the auxiliary plane will have traced on it the angle between the two planes. The hinge line PE is the intersection of the two planes POE and $PZ'E$ and the hinge line OE is the intersection of the two planes $PO'E$ and OZE. If we turn to our paper model and imagine a plane passed through the line Z_1AZ', which is at right angles to PE, the intersection of the two planes PEO and PEZ', and if we cut away all the unnecessary parts of the auxiliary plane Z_1AZ' we would be left with a triangle which would just fit between the planes PEO and PEZ', one angle of which would represent the solid angle required, *viz.*, P.

Such a triangle can be constructed as follows:—

Since the three sides are known, *viz.*, AZ_1, AZ and ZZ', the height equal to ZZ'' and the angle at Z is a right angle since Z' is directly above Z, hence construct such a triangle AZZ_3 and similarly BZZ_4.

The angle ZAZ_3 will be equal to the angle between the planes POE and $PZ'E = P$, and the angle Z_4BZ will be equal to the angle between the planes POE and $OZ'E = O$.

Cut along the lines AZ_3, Z_3Z, BZ_4 and Z_4Z, and with AZ and BZ as hinge lines, rotate the triangles into position and it will be found that Z_3 Z_4 Z_1 and Z_2 all coincide and are, as mentioned before, Z' the elevation of Z.

Next to find the third angle, that is, the angle Z between the planes $PZ'E$ and $OZ'E$ the intersection of which is EZ'.

By the rule given we can find the angle by passing an auxiliary plane at right angles to EZ'. Let this plane cut EZ' at Z' and the elevation of such a plane will be $Z'C'$ drawn at right angles to $E'Z'$ the elevation of the intersecting line. Project C' down to meet EA and EB produced in C and D respectively. Then CD will represent the intersection of the plane at the level of the sphere's centre also CD will be a hinge line and C and D will be the feet of the angle required. To find its true shape or value it is necessary to rotate it down on to the H.P. Therefore with C' as centre and radius $C'Z'$ draw a circle cutting $X'Y'$ at Z'_6 or Z'_5. Project Z'_6 or Z'_5 down to the intersection line EZ or EZ produced and join CZ_6, DZ_6 also CZ_5 and EZ_5 and the angles CZ_6D and CZ_5D will each equal the solid angle Z.

This angle can be found by another method and to prove the construction Through Z_1 and Z_2 draw two tangents at rights angles to EZ_1 and EZ_2 respectively: these tangents will intersect EP and EO produced in C and D. With C as centre and CZ_1 as radius (since CZ_1 is the true length of CZ) draw an arc, and with D as centre and DZ_2 as radius draw an arc. These arcs will intersect at Z_6 and Z_5.

Now take the perspective view, *Fig.* 2. Here again we have the solid triangle $POZ'E$ lying on the plane POE. From Z' let drop a perpendicular to Z on the H.P. or the plane POE so that Z becomes the plan of Z'. From Z draw perpendiculars ZB and ZA to EO and EP respectively.

Note.—Compare, in the previous problem and model, the triangles AZ_3Z and BZ_4Z and they will be found to be similar to $Z'ZB$ and $Z'ZA$.

By construction
$$\begin{aligned}(EZ')^2 &= (ZZ')^2 + (EZ)^2 \\ &= (AZ')^2 - (AZ)^2 + (AZ)^2 + (AE)^2 \\ &= (AZ')^2 + (AE)^2\end{aligned}$$

that is, the angle $Z'AE$ is a right angle and the angle ZAE is a right angle by construction, and therefore the angle $Z'AZ$ is the angle between the planes POE and $Z'PE$, and is the angle P.

Similarly, the angle $Z'BZ$ is the angle between the planes POE and $Z'OE$ and is the angle O.

55. Thus in both figures—

$$\frac{\sin P}{\sin O} = \frac{\frac{ZZ'}{AZ'}}{\frac{ZZ'}{BZ'}} = \frac{BZ'}{AZ'} = \frac{\frac{BZ'}{EZ'}}{\frac{AZ'}{EZ'}} = \frac{\sin p}{\sin o}$$

and similarly $\frac{\sin P}{\sin Z} = \frac{\sin p}{\sin z}$

and therefore $\sin P : \sin O : \sin Z :: \sin p : \sin o : \sin z$......(1).

and from this we get the rule that the *sines of the angles of a spherical triangle are in proportion to the sines of the sides to which they are opposite.*

56. **Given the three sides to find the value of the angles.**

Draw ZK parallel to BE and AL parallel to BZ and let ZK intersect AL at K.

Then because AL is parallel to ZB and ZK is parallel to BL, the angle ZBL = angle ALE = 90° and therefore the angle AEL + the angle LAE = 90°.

But the angle ZAE = 90° by construction.

$\therefore$ the angle LAE + z = angle LAE + angle ZAK

$\therefore$ z = angle ZAK.

$$\text{Now } \cos p = \frac{EB}{EZ'} = \frac{EL}{EZ'} + \frac{LB}{EZ'}$$

$$= \frac{EL}{EA} \times \frac{EA}{EZ'} + \frac{ZK}{AZ} \times \frac{AZ}{AZ'} \times \frac{AZ'}{EZ'}$$

$$= \cos z \cos o + \sin z \cos P \sin o$$

$$\left.\begin{aligned} \therefore \cos P &= \frac{\cos p - \cos o \cos z}{\sin o \sin z} \\ \text{and similarly} \quad \cos O &= \frac{\cos o - \cos p \cos z}{\sin p \sin z} \\ \text{and} \quad \cos Z &= \frac{\cos z - \cos p \cos o}{\sin p \sin o} \end{aligned}\right\} \text{.............................(2).}$$

57. **Given two sides and the included angle or two angles and the included side to find the remaining functions.**

To prove that $\cot O \sin P = \cot o \sin z - \cos P \cancel{\sin} z$ cos.

$$\cot O \sin P = \frac{BZ}{ZZ'} \times \frac{ZZ'}{AZ'}$$

$$= \frac{BZ}{AZ'}$$

$$= \frac{AL}{AZ'} - \frac{AK}{AZ'}$$

$$= \frac{AL}{AE} \times \frac{AE}{AZ'} - \frac{AK}{AZ} \times \frac{AZ}{AZ'}$$

$$\left.\begin{aligned} &= \sin z \cot o - \cos z \cos P \\ \text{similarly } \cot P \sin O &= \cot p \sin z - \cos O \cos z \\ \text{and } \cot P \sin Z &= \cot p \sin o - \cos Z \cos o \\ \text{etc.,} \qquad & \qquad \text{etc.,} \end{aligned}\right\} \text{.........(3).}$$

58. And if three angles are given, it can be proved that

$$\left.\begin{aligned}\cos p &= \frac{\cos P + \cos O \cos Z}{\sin O \sin Z}\\ \cos z &= \frac{\cos Z + \cos O \cos P}{\sin O \sin P}\\ \cos o &= \frac{\cos O + \cos P \cos Z}{\sin P \sin Z}\end{aligned}\right\} \qquad \text{..........................}(4).$$

59. The above formulæ, with the exception of 1, are not adapted to logarithmic computation but they can be changed to meet the case.

In formula (2) $\cos P = \dfrac{\cos p - \cos o \cos z}{\sin o \sin z}$

$$\therefore\ 1 - \cos P = 1 - \frac{\cos p - \cos o \cos z}{\sin o \sin z}$$

$$= \frac{\sin o \sin z + \cos o \cos z - \cos p}{\sin o \sin z}$$

$$= \frac{\cos (o - z) - \cos p}{\sin o \sin z}$$

$$= \frac{2 \sin \dfrac{(o - z + p)}{2} \sin \dfrac{(p - o + z)}{2}}{\sin o \sin z}$$

Now put $s = \dfrac{o + p + z}{2}$ and $1 - \cos P = 2 \sin^2 \dfrac{P}{2}$

Then $2 \sin^2 \dfrac{P}{2} = \dfrac{2 \sin (s - z) \sin (s - o)}{\sin o \sin z}$

or $\sin^2 \dfrac{P}{2} = \dfrac{\sin (s - z) \sin (s - o)}{\sin o \sin z}$.

Similarly $\cos^2 \dfrac{P}{2} = \dfrac{\sin s \sin (s - p)}{\sin o \sin z}$

$$\therefore\ \tan^2 \frac{P}{2} = \frac{\sin (s - z) \sin (s - o)}{\sin s \sin (s - p)} \text{ ....}(5)$$

and similarly $\tan^2 \dfrac{Z}{2} = \dfrac{\sin (s - p) \sin (s - o)}{\sin s \sin (s - z)}$ ....(6).

60. As the angle P at the pole is the *hour* angle of a heavenly body formula (5) is used for *Time* computations, and as the angle Z at the Zenith is the *azimuthal* angle of a heavenly body formula (6) is used for *Azimuth* computations.

The four general equations established in formulæ (1) to (4) are sufficient for the solution of right-angled triangles, and the practical use of such is when observations are taken to circumpolar stars at Elongation, or when the angle at the star or angle O, known as the parallactic angle is 90°.

In formula (1) let O = 90° and therefore sin O = 1

then $\sin P = \dfrac{\sin p}{\sin o}$ or $\sin p = \sin o \sin P$ ....(7).

and $\sin Z = \dfrac{\sin z}{\sin o}$ or $\sin z = \sin o \sin Z$ ....(8).

In formula 2 let O = 90° and therefore cos O = O

$\therefore\ \cos o = \cos p \cos z$ ....(9).

In formula 3 with the same arguments—

$\cot P = \cot p \sin z$ ....(10).

$\tan p = \tan P \sin z$ ....(11).

$\tan z = \tan Z \sin p$ ....(12).

$\tan z = \cos P \tan o$ ....(13).

$\tan p = \cos Z \tan o$ ....(14).

In formula 4 with the same arguments—

$\cos p \sin Z = \cos P$ ....(15)

$\cos z \sin P = \cos Z$ ....(16).

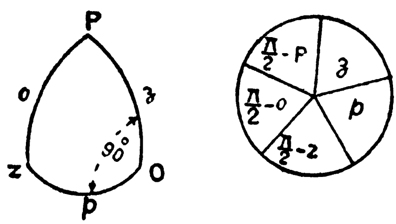
61. The formulæ 7 to 16 are best numbered by Napier's rules of *circular parts* which are as follows:—

If the right angle is neglected then there are 5 parts remaining and these are the two sides containing the right angle, the complement of the side opposite the right angle and the complements of the two angles.

These five circular parts are written in the order in which they occur in the triangle, *viz.*, $\frac{\pi}{2} - P, \frac{\pi}{2} - o, \frac{\pi}{2} - Z$ and p since O is the right angle; any three circular parts are taken, one of them can be so chosen that the other two are either both adjacent or else both opposite to it.

The diagrams show how the parts are written (*Fig.* 13).

Fig. 13.



The selected part is termed the *middle* part and Napier's rules are as follows:—

1. *Sine* of the middle part equals the *product* of the *tangents* of the *adjacent* parts.

2. *Sine* of the middle part equals the *product* of the *cosines* of the opposite parts.

Example by rule 1—

$\sin p = \tan(\frac{\pi}{2} - Z) \tan z = \cot Z \tan z$ [compare (12) *supra*].

By rule 2—

$\sin p = \cos(\frac{\pi}{2} - P) \cos(\frac{\pi}{2} - o) = \sin P \sin o$ [compare (7) *supra*].

etc., etc.,

Astronomy—Definitions.

62. Reference has already been made to the spherical triangle PZO, and in astronomy, for the purpose of the surveyor, this has a special

value, and is the triangle formed in the celestial sphere by O the object, P the pole and Z the zenith.

As O the object is a moving object (sun, moon, planet or star) the conditions of the triangle in question differ at every instant. Astronomers speak of the object as moving, whereas it is the earth which is revolving and rotating, and it is for this reason that the expression "apparent" is used to denote that the earth is accepted as stationary and that celestial objects are moving. Astronomy is based on this supposition.

Definitions are given in all handbooks and works on Astronomy, and reference should be made to them where necessary, but a few of the most useful will be explained so that the reader may be familiar with the arcs and angles with which he will have to deal.

The best way to understand such definitions is for him to stand in an open space on a clear night and to imagine the celestial hemisphere as a vast dome on which different celestial objects have been impinged.

Supposing he is in the northern hemisphere the first star he must recognise is Polaris or α Ursæ Minoris (*see* para. 67 on Azimuth). This star is very near the north pole and is gradually getting nearer; it is about 1° away or its N. P. D. or north polar distance is 1°.

For the purpose of demonstration let the observer accept Polaris as one of the poles of the earth's axis (PP Plate IX), that is, points in the celestial sphere in direct prolongation of the earth's axis.

The point immediately above the observer's head is his *Zenith* and opposite to Zenith is *Nadir* (denoted by Z and N). We have now the poles, zenith and nadir.

The plane of the great circle, at right angles to the poles or axis of the earth, and passing through the earth's centre, is known as the *celestial equator*—SS, which is therefore coincident with the earth's equator; the plane at right angles to Z and N is the celestial horizon HH which passes through E the earth's centre, but since the earth in astronomy is in most cases* considered as a point in space, the celestial horizon may be called the horizon.

Great circles passing through the poles PP will cut the celestial equator SS at right angles and are known as *Declination* circles, *viz.*, POP

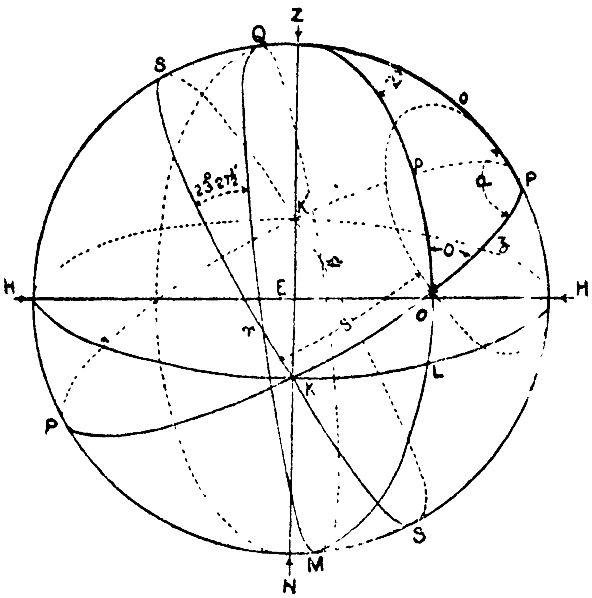
Great circles passing through Z and N will cut the celestial horizon at right angles and are known as *Vertical* circles, *viz.*, ZON.

The great circle passing through ZPN is the only instance when a vertical circle and a declination circle coincide and is called the *Meridian* HPZSH. The meridian of the observer is therefore the great circle

*A point on its surface is only considered when near objects such as the Sun, Moon and Planets are observed to.

CHAPTER III.

Plate IX.



[To face page 69.

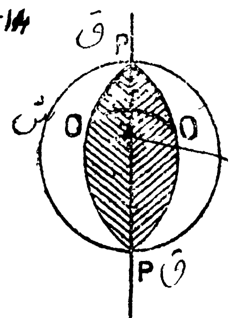
passing through his zenith and the poles, and this circle cuts the horizon in N and S points, and a great circle passing through his zenith at right angles to his meridian will be the *Prime vertical* ZKN.* This prime vertical cuts the horizon in the observer's east and west points K and K′.

Now let us consider an object O travelling in its orbit around P (shown in dotted lines); the declination circle through O is POP and the declination of the star is OK, in this case *north* declination, so that *declination* may be defined as that portion of the arc of a declination circle passing through the object intercepted between the object and the celestial equator, usually known as δ. If the object is north of the celestial equator it is said to have declination N, and if south, declination S. The complement of the declination is the *north polar distance*, usually written N. P. D, that is, $90° - \delta$ = PO or NPD = PO = z if the star has a declination N, and is $90° + \delta$ if the star has a declination S.

The declinations of the sun, moon, planets and a certain number of stars, are found in the Nautical Almanac (N.A.)

The *hour angle* of an object is the angle between the plane of the meridian at the place and the plane of the declination circle passing through the object. In figure ~~23~~ 14 it is the angle ZPO or P, and is that part of the arc of the celestial equator intercepted between the meridian and the declination circle passing through the object.

Fig. 14.



Its measure is given in hours, minutes and seconds or in degrees, minutes and seconds and is W or E according as the hour angle is W or E of the meridian.

It will be noticed that as O the object is moving, if in the east it is rising and the hour angle is decreasing, and when it is in the west it is setting and the hour angle is increasing. When the object arrives on the meridian (transits) its hour angle is zero or it vanishes. Again, if the value of the hour angle at a given instant is known and the declination also, the position of the object is found and this is one system of *celestial co-ordinates*, *viz.*, hour angle and declination.

The declination of a star changes very gradually and is scarcely appreciable except every five days or so, but that of the sun, moon and planets is appreciable for short periods of time and a reference to the N.A. will make this clear.

* The reader should note that some stars (according to the observer's latitude) will never cross the Prime Vertical and only at the equator is it possible for a star to have a course coincident with the Prime Vertical and the declination of such a star would be 0° 0′ 0″

Again, with O the object in the east, it is rising, and therefore its altitude or distance from the horizon is increasing or its zenith distance is decreasing until it *transits*, and its altitude then decreases and its zenith distance increases as it sets into the west. *Altitude** may be defined as that portion of the arc of a vertical circle intercepted between the object and the celestial horizon as OL and its complement is ZO or zenith distance $= 90° - OL$.

Again, with the same argument as with the hour angle at P, the angle OZP or Z decreases till the object arrives on the meridian and thereafter increases as the object sets. This angle, between the plane of the meridian of the place and the plane of the vertical circle passing through the object, is known as the *azimuthal angle* or *Z*, and this angle is also measured as that portion of the horizon HH intercepted between the vertical circle passing through the object and the meridian of the place, as LH, and is measured in degrees, minutes and seconds.

If the azimuthal angle and the altitude of the object are known then the position of the object is known at that instant and this is another system of *celestial co-ordinates*.

When the object O arrives on the meridian it is said to *culminate*, and it culminates twice in a *sidereal* day, once on the meridian above our heads known as the *upper* culmination (better known as transit) when by the foregoing arguments its hour angle and azimuthal angle are *nil*, and at its *lower* culmination, or when it arrives at the meridian below our feet, when its azimuthal angle is *nil* and its hour angle is 180° or 12 hours in *sidereal* time.

The *amplitude* of a heavenly body is the angle measured along the horizon between the vertical circle passing through the object and the E and W points.

The latitude (λ) of a place is the inclination of the normal or plumb-bob to the plane of the equator and is measured along the meridian and is ZS or the angle between the planes SKS and ZKN.

Now $SP = 90°$ also $ZH = 90°$ $\therefore$ $ZS = PH$, or the altitude of the elevated pole is the latitude of the place, and ZP its complement is the co-latitude of the place or *o* of the spherical triangle ZPO.

Therefore, given the latitude (λ) of a place which can be found on a map and need not be known with any great accuracy for time and azimuth results to say 5 to 10 secs., and since the declination δ and hence the N.P.D. can be found in the N.A., if we observe OL we know the three sides of the spherical triangle; for ZP or *o* is the co-latitude

* Altitude means observed altitude corrected for parallax and refraction.

and ZO or p is the co-altitude and PO or z is the N.P.D. thus by the

formulæ $\tan^2 \frac{P}{2} = \frac{\sin(s-z)\sin(s-o)}{\sin s \sin(s-p)}$ ....(5).

and $\tan^2 \frac{Z}{2} = \frac{\sin(s-p)\sin(s-o)}{\sin s \sin(s-z)}$ ....(6).

that is, that the hour angle P and the azimuth angle Z can be computed.

If O an object has z its N.P.D. less than PH, then it will be seen that it will always be above the horizon HH and the object (always a star) is then known as being *circumpolar*. A circumpolar star is thus defined as one whose N.P.D. is less than the latitude of the place or its N. declination is greater than the co-latitude of the place. With reference to the apparent motion of stars it should be noticed that those on the prime vertical ZKN will be moving faster than an object off the prime vertical since it has a greater arc to traverse in the given time and hence to obtain good *time* results an object on or as near as possible to the prime vertical should be chosen.

The *ecliptic* is a great circle along which the sun travels annually among the stars or is the apparent motion of the sun among the stars and intersects the celestial equator obliquely as QM. The angle that the plane QM makes with the celestial equator SS is 23° 27½′, and this angle is known as the *obliquity of the ecliptic;* * the two intersection points are known as the *equinoxes*.

When the travel of the sun or the sun's apparent motion among the stars is from south to north of the celestial equator the point of intersection is known as the spring or *vernal equinox* or the first point of Aries (♈), and when the sun seems to travel from north to south the point of intersection of the celestial equator is known as the *autumnal equinox* of the first point in Libra. The reader will understand that at the two equinoxes the sun's declination is *nil*.

The Declination circle passing through the Equinoctial points is known as the Equinoctial Colure.

The *zodiac* † is a zone extending about 8° on each side of the ecliptic and is divided into 12 equal parts, each of 30°, which are known as the signs of the zodiac. The names of the signs are as follows :—

مینڈھا برج ثور بیل برج جوزا برج سرطان کیکڑا

حمل Aries (ram), Taurus (bull), Gemini (heavenly twins), Cancer (crab),

* So called as the ancients noticed that the eclipses of the moon took place on this circle.

† The ancients calculated that the earth returned to the same position among the stars in 360 days. They gave 12 Signs to the Zodiac to portion off 12 months. As the diameter of the moon or sun seems to be ½° (really 32 minutes) or the sun travelled a double minute in its course of one diameter we get 30° for each sign of the Zodiac and 360° for the whole circle and 60 minutes and 60 seconds as divisions in time.

Leo. (lion), Virgo (virgin), Libra (scales), Scorpio (scorpion), Sagittarius (archer), Capricornus (he-goat), Aquarius (water-carrier), Pisces (fishes). Two of the signs include the equinoxes—Aries and Libra.

The *first point of Aries* occurs on or about the March 22nd of each year and the autumnal equinox about the 23rd September of each year, and midway between these dates the sun will have reached either its most northern declination of 23° 27½′ (N) known as the summer solstice or its most southern declination of 23° 27½ (S) known as the winter solstice. The term solstice is derived from *Sol* (Sun) and *sto* (stop or stand) meaning where the sun seems to stop or halt in its passage northwards or southwards.

The term summer solstice as we understand it applies only to the Northern hemisphere which is in reality the winter solstice of the Southern hemisphere, and so also the vernal or spring equinox will be an equinox occurring in the autumn, say in Australia or New Zealand.

Now sun time and star time are based on the first point of Aries, and it would not be unprofitable to explain how this imaginary point in the heavens derived its name.

The first point of Aries was a star in the constellation of the Ram about 3,000 years ago, that is, the vernal equinox took place at that time at or very close to a star in Aries, and this was when Astronomy was in its infancy or became known as science. Since then it has moved and the imaginary point is now in Andromeda and is slowly moving towards Hercules. This movement is due to the earth being an oblate spheroid and to the sun and moon exerting an unequal attraction on the earth's inequality and each tending to drag the earth into its own orbit, the result being a backward or retrograde motion of the intersection of the ecliptic and celestial equator along the celestial equator, that is, that the first point of Aries retrogrades along the equator and this retrogression is about 50·1″ a year and is known as the *Precession of the Equinoxes*. This Precession of the Equinoxes was discovered by the Greek Astronomer Hipparchus, and has been calculated to take 25,868 years to encompass the Zodiac or the sun's apparent motion among the stars.

The forces which cause precession do not act uniformly so there is a slight wobble in the axis and the direction of the axis and the pole is not stable. The total difference at the N. pole is roughly, a square of 50 feet. This is called *Nutation* from *nutare* (to nod).

Aberration is the error in the apparent place of a body produced by the earth's motion in a direction contrary to that of the ray of light from the body.

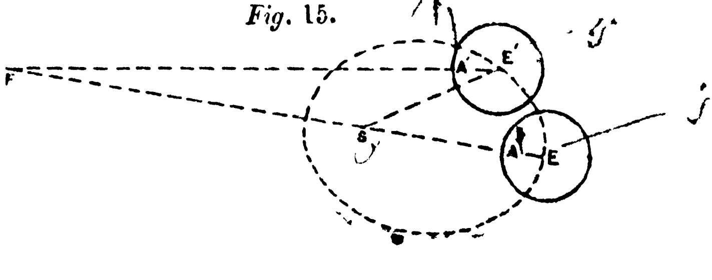
It is necessary now that the reader should understand that there are three kinds of time, *viz.*, Star time or *sidereal* ~~time~~, Sun or *apparent* time or time kept by the true Sun which is sun-dial time, and *mean* time or the time kept by well regulated clocks and watches, and all these times have as their zero the first point of Aries since all ***Right Ascensions*** (R.A.[s]) are reckoned eastwards from this one point. The right ascension of an object is the angle between the plane of the declination circle passing through the object and the plane of the declination circle passing through the vernal equinox or, in other words, is that portion of the arc of the celestial equator intercepted between the declination circle passing through the object and the first point of Aries. The angle ♈PK or the arc K♈ represents the R.A. of O. Right ascensions are reckoned from west to east from zero to 360° or from 0 hour to 24 hours. For example, if ♈K in figure was 15° its R.A. would be one hour, but if K had been 15° on the other side of ♈ as at K′ then its R.A. would have been 345° or 23 hours (behind the first point of Aries). Right ascension is like terrestial longitude except that terrestial longitude is reckoned east and west of Greenwich 180°. K in figures would be 1° W. Longitude and K′ would be 1° E. Longitude.

63. It has been explained how the first point of Aries governs our time, and we may proceed to distinguish between the different times.

A star will cross the meridian at exact intervals of time, so let us suppose we set up a theodolite and sight the vertical wire on the meridian of the place and note the time that a particular star transits and for days continue doing this noting the time; if our time-keeper showed exact intervals of 24 hours then the time kept would be sidereal time and our time keeper would be a sidereal clock, but if the time-keeper was a mean time or ordinary ~~watch~~ (regulated to keep true mean time) then the star would cross the meridian or transit also at exact intervals of time but 3 minutes 56 seconds (nearly) earlier every day, and this would happen day after day till 24 hours would have been gained by the star on mean time in a *sidereal* year. The reason for this is as follows:—

In *Fig.* 15 suppose E to be the position of the earth in its orbit

Fig. 15.



when it is noon at A, or a transit of the Sun S is occurring, and let F be a fixed star in the same line EAS, produced to an infinite distance of the earth from the sun.

Then suppose that in the time that the earth has made one complete revolution on its own axis, it has moved along its orbit round the sun into the position E′. At this moment the fixed star F will make another transit, since EE′ is nothing in comparison to the distance EF, and the angle EFE′ is inappreciable, so that E′ A′ is really parallel to EA.

But quite the contrary is the case at this moment with the Sun; here the distance EE′ is quite appreciable in comparison with ES, and the angle ESE′ is quite measureable, and it will take the earth nearly four minutes more to revolve through the angle A′E′S, when a transit of the sun will occur.

Hence a solar day, which is the interval between two transits of the sun, is four minutes longer than a sidereal day, or the interval between two transits of a fixed star, and that if the earth in its revolution on its orbit around the sun makes one rotation less in the interval so that if this interval is a year, a *tropical year* may be defined as the time in which the sun moves from vernal equinox to that point again, and a sidereal year is the time in which the sun moves from a fixed star to the same star again, or the time it takes to perform an absolute revolution and returns to the same position among the constellations. It must be remembered that owing to the precession of the equinoxes every year the sun does not return to the same point it started from; and Bessell has calculated that a *mean solar* or tropical year is 365·2422 mean solar days, and as the sidereal year is one day more

∴ 366·2422 sidereal days = 365·2422 mean solar days

and 1 sidereal day = 0·99726957 mean solar day,

or 24 hours sidereal time = 23 hrs. 56 mins. 4·091 secs. of mean time or the daily *acceleration* of sidereal time (S.T.) on meantime (M.T.) is 3 mins. 56 secs. approximately, and the *retardation* of M.T. on S.T. is 3 mins. 56 secs. approximately, or acceleration and retardation at 9·8565* seconds per hour (usually taken as 9·86).

As there are roughly 365¼ days in a tropical year and as a quarter of a day cannot suitably be made provision for, the years are made up of an even number of days known as equatorial years, and Julius Cæsar arranged that every year should be a 365-day year except the 4th year or year divisible by 4 which would be a leap year or a year of 366 days.

This calendar is known as the *Julian Calendar*. This rather over-

* *See* Tables in N.A. and Chambers's Mathematical Tables, page 433, latest edition.

corrects the error, and it has been calculated that a day is gained in every 100 years, so the century year is not a leap year except as follows:—that as this rather undercorrects the error and a day is lost in 400 years Pope Gregory inaugurated that every century year divisible by 400 should be a leap year. Thus 1896, 1904 and 2000 are leap years and 1900 not. This is known as the *Gregorian Calendar.**

64. It has been noted that the vernal equinox occurs at or about the 21st March each year or when the declination of the true sun is 0° that is when the true sun is on the Celestial Equator.

The RA of the apparent or true sun at this instant will be $0^h. 0^m. 0^s.$; but since we are dealing with a mean sun the Sidereal Time of G. M. N. for the date will differ with the R.A. of the true sun by an amount equal to the Equation of Time in Sidereal units. The following extract from the N.A. 1925 will make this clear—

Date March.	Apparent R.A.	Apparent declination.	Equation of Time.	S.T. at GMN
	h. m. s.		m. s.	h. m. s.
20	23 57 41·41	S_0—15-01·2	7 41·5	23 49 59·92
21	00 01 20·06	N_0—08-41·1	7 23·6	23 53 56·47

On March 22nd by argument already given, the sun will cross the meridian at 12 noon by the mean time clock, but the sidereal clock will have gained 3 mins. 56 secs. and the sidereal clock will be 12 hours ahead at the Autumn equinox, and so on, hence we get in the N. A. the sidereal time at Greenwich Mean Noon (S.T. at G.M.N.) or in other words the *hour angle of the sun* at Greenwich. These sidereal times are tabulated for different days in the year because all R.As. of celestial objects are reckoned from the first point of Aries.†

Next, if an observation is taken E. or W. of the Greenwich meridian or the meridian adopted by the N.A. and most nations have adopted this zero, it must happen that the first point of Aries will transit in one case at Local Mean Noon (L.M.N.) before Greenwich and in the other case after, so that if in 360° there is a change of 3 mins. 56 secs. in sidereal time then a proportionate correction *minus* when east and *plus* when west must be applied to the sidereal time of G.M.N. to reduce it to S.T. at L M.N.

* If the year divisible by 128 is not considered a leap year, there would be an error amounting to a day in 100,000 years.

† The reader will understand that if the sidereal clock at Greenwich beats 0 hours when the declination circle passing through the first point of Aries coincides with the vertical wire of the meridian that if he noted the times of certain celestial objects one after the other crossing the wire he would be tabulating a series of Right Ascensions (in sidereal time) of celestial objects Again, if the instrument had been set with, we will say, a vertical are graduated at 0° when the telescope intersected the first point of Aries on March the 22nd (then on the celestial equator) then as stars transited and their elevations or depressions were registered with reference to this 0° line exclusive of correction for refraction he would obtain the declinations N or S, as they appeared above or below this zero-plane

Example.—The S.T. for G.M.N. on a certain date was found to be 12 hours ; what is the S.T. at L.M.N. at a place 90° W. and E. Longitude ?

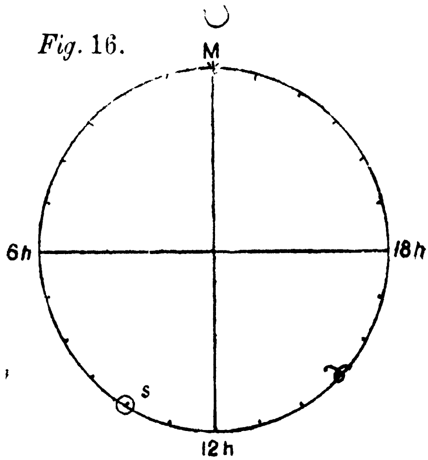
$90° = \frac{360°}{4}$ ∴ a correction of $\frac{3'\cdot 56''}{4}$ is to be applied.

Thus the S.T. at L.M.N. for 90° W. = 12 hrs. + 59 secs. = 12 h. 0 m. 59 s., and the S.T. at L.M.N. for 90° E. = 12 hrs. – 59 secs. = 11 h. 59 m. 01 s.

It is necessary now to show if the meridian at a place is known how the L.M.T. is found. This is best done by a diagram and an example.

Example.—Given the S. T. at G.M.N. for a certain date, as 5 hrs. 0 min. 51 secs. and the R. A. of a star as 15 hours ; find what the L.M.T. is at a place when the star is at transit, the longitude of the place being 77° 54′ E.

Fig. 16.



Let the diagram *Fig.* 16 show a vertical section through the celestial sphere and let it be divided into four quadrants of 6 hours each numbered anti-clockwise and let 0 hours or 24 hours be the meridian.

When the particular star is on the meridian its position is known so on the diagram it is placed at 0 hours or at M. Since its R.A. is 15 hours we can measure back 15 hours and place the first point of Aries on the diagram because 15 hours have elapsed since the first point of Aries was on the meridian.

Again it is known, with reference to Greenwich, that the interval of time (sidereal) lapsed from the first point of Aries, when the sun was on the meridian at Greenwich, was 15 h. 0 m. 51 secs., and therefore in Longitude 77° 54′ E. the interval will be 5 h. 0 m. 51 s.–51 s. or 5 hours exactly, and so in the diagram the mean sun is placed 5 hours back from ♈. Thus it will be seen that the sun with respect to the local meridian is 10 hours ahead, that is, that 10 hours (sideral) have

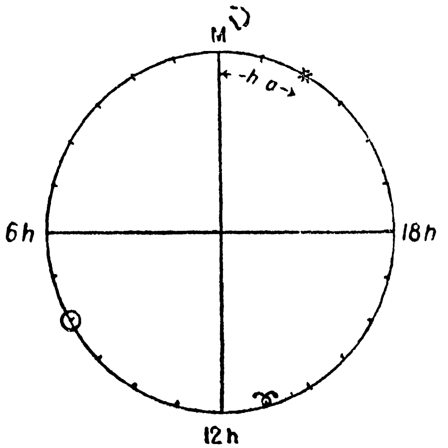
From 1925 onwards the times in the N.A. will be from midnight, *i.e.*, G.M.T. or Greenwich mean time will be reckoned from midnight and midnight on the 1st January will mean the end of the day on the night of New Year's day, that is, that midnight New Year's Eve is 0 day of the year and noon on the 2nd January will be 1·5 days.

lapsed since local noon, and to reduce this to mean time units at 9·86 secs. per hour the result 9 hours 58 minutes 21·33 seconds is obtained in mean time units, as the L.M.T. of observation.

If the meridian is not known but the hour angle had been calculated by the formula already given and found to be 2 hours with the star in the east and with the conditions given in the last example we have the following diagram *Fig.* 17 and placing.

Since the star's hour angle (*h.a.*) was 2 hours from the meridian and the star was in the E. it can be placed 2 hours to the right of M. If the star had been in the W. then the position would have been 2 hours to the left of M. the meridian. Also in the example, the first point of Aries is placed 15 hours anti-clockwise from the star and the sun 5 hours back from the first point of Aries, and the interval of L.M.N. in sidereal units is found to be 8 hours and the L.M.T. of observation would, on reduction, be found to be 7 hours 58 minutes 41·1 seconds.

Fig. 17.



A useful equation can thus be devised and this should be remembered.

R.A. (+ 24 hours if necessary) $\begin{smallmatrix}+\mathrm{W}\\-\mathrm{E}\end{smallmatrix}$ *(h. a.)* − S.T. at L.M.N. = sidereal interval (S.I.) from L.M.N.

If the result is reduced to mean time units the correct local time is obtained.

Consider the equation and suppose the time of transit of a heavenly body is required or when the *h. a.* of the body is 0; the second value of the equation vanishes and we get R.A. (± 24 hours) − S.T. at LMN = S.I. from L.M.N.

Example—When will Polaris (*a* Ursæ Minoris) *transit* in Longitude 77° 54′ E. on the 12th November, 1909. The R.A. of Polaris in the N.A. for 1909 on that day is given as 1 *h.* 27 *m.* 28 *s.* and the S.T. at G.M.N. on 12th November, 1909 is given as 15 *h.* 23 *m.* 53·7*s.* whence S.T. at L.M.N. = 15 *h.* 23 *m.* 53·7 *s.* − 51 *s.* (correction for Long. E.) = 15 *h.* 23 *m.* 02·7 *s.*; and R.A. + 24 hours = 25 *h.* 27 *m.* 28 *s.*, therefore sidereal time of transit = 25 *h.* 27 *m.* 28 *s.* − 15 *h.* 23 *m.* 02·7 *s.* = 10 *h.*

04 *m.* 25·3 *s.* in sidereal units or the local mean time of transit is 10 *h.* 02 *m.* 46·3 secs.

When clocks and watches keeping mean time are set to a certain *standard* meridian a correction must be given according to the departure E. or W. of that meridian.

This correction for *standard time* at a place 15° W. of the local standard time meridian will be minus one hour and if 15° E. plus one hour to obtain local time.

65. The time kept by the true sun is sun-dial time but this is not the time kept by watches and clocks as sun time is variable. If the orbit of the earth were a circle with the sun as centre the apparent day would be a constant interval of time, but the earth's orbit is an ellipse with the sun in one focus, and Kepler's second law proves that the earth travels through equal areas at equal intervals of time or that the radius vectors sweep out equal areas in equal times, and it thus stands to reason that the earth is travelling faster when it is nearest the sun or at *perihelion* and slower when it is at its *aphelion*. Again, the sun's path is along the ecliptic which is inclined to the celestial equator and hence a *second* variation of time.

These are the main reasons* for the sun's time being variable, and since no clocks could be regulated to go fast and slow to conform with the sun, astronomers have arranged to have a uniform day and this day to coincide more or less with day and night and therefore observations to the sun have to be reduced from what is *apparent or true* solar time to mean time ; this correction is termed the *Equation of time*, and its amount, whether additive or subtractive, will be found in the N.A., page I. of every month—the equation of time being in sidereal units.

Apparent time may be defined as the angle contained between the meridian of the place and the meridian passing through the true sun. *Mean time* as the angle contained between the meridian of the place, and the meridian passing through an imaginary sun moving on the equator with the mean velocity with which the true sun appears to move in the ecliptic. The angle contained between the meridians passing through the *true* and *imaginary* suns is the *equation of time*. Sun-dials therefore should be corrected for equation of time to conform to *local* mean time.

It is next required to interpolate the variation in 1 hour for 18 *h.* 9 *m* between the noons of the 1st and 2nd June and the variation will be found to be ·379 *s.* This variation multiplied by 18·136 hours = 6 87 *s* and since equation of time for 1st June = 2 *m.* 28·67 *s.* decreasing then

* See Barlow and Bryan's Mathematical Astronomy for further explanation.

the equation of time = 2 *m*. 28·67 *s*. – 6·87 *s*. = 2 *m*. 21·8 *s*. and therefore L.M.T. of L.A. = 12 *h*. 0 *m*. 0 *s*. – 2 *m*. 21·8 *s*. = 11 *h*. 57 *m*. 38·2 *s*. and therefore the clock for local time was 11 *h*. 57 *m*. 38·2 *s*. – 11 *h*. 35 *m*. 40 *s*. = 21 *m*. 58·2 *s*. slow. Had however the clock been keeping standard time for 5½ hours or 82½° E. of Greenwich it would have been 4 *m*. 0 *s*. + 1 *m*. 58·2 *s*. slow or 5 *m*. 58·2 *s*. slow.

The foregoing examples show how time is found by observation to a celestial body on the observer's meridian.

If the celestial body is a star the computations are greatly simplified as there is no reference to Greenwich Mean time which will be explained under the next para. It is not usual that the meridian is known, and hence observations to the sun or stars out of the meridian, or *ex*-meridian as they are sometimes called, will be next explained.

66. **Observations to the sun or stars out of the meridian for time and azimuth.***—The following are the corrections to be applied to astronomical observations and a general idea of how observations are best made :—

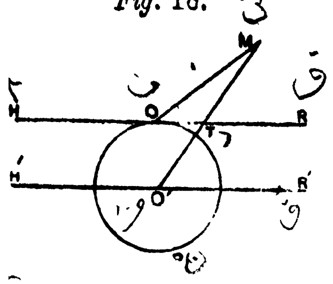
Refraction.—A ray of light from any celestial body upon entering the atmosphere becomes bent or refracted in a downward direction, and the closer the body is to the horizon the greater the width of the belt of atmosphere through which the ray has to pass and therefore the greater the refraction. Refraction thus tends to raise an object or the elevated angle read is more than the true, and thus refraction is always *subtracted* from the observed *altitude* to obtain the true altitude. Refraction is dependent on the temperature and height of the barometer, (*see* Table III., Appendix II.,) or 58″ cot altitude gives a close value.

Parallax.—In the ordinary processes of astronomy, we assume that the stars, *i.e.*, fixed stars, are at infinite distances, that is, the rays from a fixed star to the surface of the earth and also to the earth's centre are coincident, and we thus assume that the observations are made at the earth's centre and referred to a plane parallel to a plane of the sensible horizon of the observer's position. But this does not hold good with the solar system.

* The author here reiterates his caution concerning the footscrews for levelment of bubbles during a set of observations (a set includes readings to R.M.) involving *horizontal* angles. The footscrews should not be touched and any levelment of the upper level should be done by the antagonising screw (*vide* para. 18 Part II)

Let OP be a section of the earth; O any point on the surface;

Fig. 18.



HOR the horizon at that point; H′O′R′ the rational horizon through the centre of the earth and parallel to HOR. Let M be the sun, moon, or some planet, then MOR is the altitude of the body above the horizon HOR, but MO′R′ (= MTR) is the altitude of the body above the rational horizon *i.e.*, the altitude above the horizon through the datum point. Now MTR = MOR + OMT, *i.e.*, the true altitude is greater than the observed altitude by the angle OMT, which is called the *parallax*. It is evident from the figure that the magnitude of this angle OMT depends on the altitude of M, being nothing in the zenith, and attaining its maximum on the horizon. The fixed stars are so far from the earth that the magnitude of this angle is imperceptible, and even the horizontal parallax from the sun never exceeds 9 seconds, so therefore if observations are being made with an instrument which only reads to minutes, as, for instance, the ~~pocket sextant~~, the correction for parallax may be wholly omitted. Correction for parallax must *be added to* observed altitudes or *subtracted from* zenith distances, *vide* Table IV., Appendix II.

Semi-diameter.—When an observation has to be made on the sun by means of an alt-azimuth instrument, it would be very difficult exactly to bisect the disc of the sun by the horizontal cross wire of the instrument, so it is usual to read the elevation of one of the limbs, either the upper or lower, and then, as the case may be, subtract or add the semi-diameter of the sun, to find the true altitude of the centre. This is called the *correction for semi-diameter*, and is given in the "Nautical Almanac" for every day in the year. This correction is in reality the angle subtended at the eye of an observer placed in the centre of the earth by the semi-diameter of the sun, and is constantly, though slightly, varying; not because the diameter of the sun varies, but because the distance between the sun and the earth is ever changing. Also, when a sextant and an artificial horizon are used, the altitude of one of the limbs instead of the centre is usually read, because the observer can note with much greater accuracy when the two Suns just touch one another than when they exactly coincide.

To all observations for altitude, after having observed the altitude of the sun and the readings of the barometer and thermometer at the

time of observation, the corrections above-mentioned should be applied in the following order :—

First make the corrections due to instrumental errors; then take out of Table III. the refraction due to the observed zenith distance corrected for instrument. Now this number was calculated at an assumed temperature of 50° F., and a barometric pressure of 30 inches. Table III. gives the necessary corrections with their signs. Then having corrected the refraction, add it to the observed Z.D. Then apply the semi-diameter (taken from the "Nautical Almanac"), subtracting it if the observation was made on the lower limb* and adding if on the upper limb; and finally take out the correction for parallax and subtract it from the Z.D.

Example 1.—The observed altitude of the sun's upper limb is 39° 16′ 20″ at 8 A.M. on June 20th, 1922; barometer, 28·85 inches; thermometer, 80° F., no error for instrument; find the true altitude of the sun's centre.

Observed Z.D. (90° – alt.)		50° 43′	40″
Refraction for 50° 0″ 0″	+	1	9·4
And change for 43′ 40″	+		1·8
Correction for barometer, 28·85 inches	–		2·7
„ „ thermometer, 80°	–		4·2
Value of corrections	+	1	04·3
∴ corrected refraction for 50° 43′ 40″ is	+	1	04·3
Semi-diameter (from N.A.)	+	15	46·3
Parallax in altitude	–		06·5

True altitude of sun's centre (90° – Z.D) = 38° 59′ 36″

Example 2.—The observed double altitude of the sun's lower limb at 8 A.M. June 22nd, 1922, was 84° 44′ 40″; index error, 3′ 45″ *on the arc* (therefore negative); barometer, 28·85 inches; thermometer, 85° F.: find the true altitude of the sun's centre.

Observed double altitude	84°	44′	40″
Index error	–	03	45
	2)84	40	55

Single altitude		42° 20′ 27·5″ = 47° 39′ 32·5″ Z.D.
Refraction for 47°	+	1′ 02·4″.
∴ change in 39′ 32·5″	+	1·46″
Correction for barometer, 28·85	–	2·44
„ „ thermometer, 85°	–	4·38
Value of corrections		57·0″
∴ corrected refraction for 17° 39′ 32·5″ is	+	00′ 57·0″
Semi-diameter, June 23rd	–	15′ 46·2″
Parallax in altitude	–	6·4″

True altitude of sun's centre 90° – Z.D. = 42° 35′ 23″

*Note that the object as viewed through the telescope of an ordinary theodolite [without using the inverting eye-piece] is reversed.

In the above there appears to be a certain amount of unnecessary labour in calculating refractions from a Table of Zenith Distances, but the system has been adopted since most refraction tables used for astronomical work are for zenith distances and not for altitudes. The reader will now understand that when dealing with celestial objects other than fixed stars he has to apply the following corrections:—

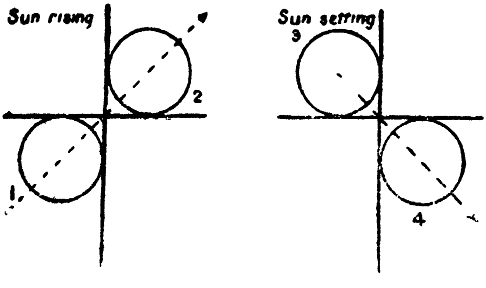
Right ascensions and Declinations with reference to their position, at the same instant, to be very exact, at Greenwich, and the correction for parallax. Correction for refraction is common to both. Correction for semi-diameter is for the sun only, but it is advised that the limbs of the sun be observed to in preference to the centre which cannot always be accurately intersected and requires a great deal of judgment in any case.

Thus for time observations it is recommended to allow one limb of the sun to cross the horizontal wire and register the time of contact and then permit the sun to cross the wire (steadily working the horizontal slow-motion screw of the plate in case he has the tendency to move out of the field) and then to register the time of the second contact or the final crossing of the sun across the horizontal wire. In this way, one vertical angle is recorded and two times, and the mean of the two times is the correct time of the intersection of the sun's centre at the vertical angle observed, and the time of the sun's centre is connected to the clock which is to be examined for the local mean time or standard time it is keeping *see* example, page 95.

67. **Azimuth.**—For azimuth or the direction of the meridian and when dealing with the sun *ex*-meridian, the computed angle Z is connected to a point on the earth's surface as a referring mark which plays the same part as the clock does for time observations. Hence, to obtain an azimuth from the sun, correct time not being known, the horizontal angle to a referring mark (R.M.) and altitude of the sun's centre is required; the following method might be adopted:—

In latitudes greater than 23° 27 N. the sun will always rise south of the east point and will have a motion when rising and setting, of left to right when the observer is facing south, and the motion will be as shown in diagram *Fig.* 19, though, when looking through the telescope, it will be reversed. What the observer must do, after determining the direction of the sun's motion in the telescope, is to bring one quadrant, formed by the horizontal and vertical wires in contact with the sun's limbs and to note he time, horizontal and vertical angles, taking care that the theodolite is in true levelment. The standard time corrected to local

Fig. 19.

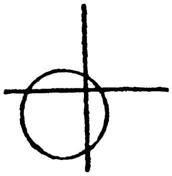


*Corrections for Barometer and thermometer are only necessary for Latitude Observations.

time and referred to Greenwich need not be very exact but sufficiently so in order that the sun's declination can be calculated to the requirements of the accuracy needed. A small error in the declination will not affect the ultimate result materially. He next proceeds to get a contact with the opposite quadrant and registers the time, horizontal and vertical angles. The mean of the times of observation and mean angles will give the time of the observed altitude of the sun's centre and the horizontal angle to it.

After obtaining the first contact, say in position 2 and recording time and angles, there is nothing gained in moving both the horizontal and vertical arcs to make the second contact, in fact, it is better to leave the vertical arc alone and to follow the sun with the horizontal slow-motion screw, and after a little lapse of time if the sun is rising he will be in a position, as nearly as possible as shown in diagram *Fig.* 20. The observer should now take care and keep as much of the sun to the right of the vertical wire as there is above the horizontal wire or, in other words, he should endeavour to cut equal segments of the sun with his 3rd quadrant and in this manner, by a final manipulation of the horizontal tangent screw or slow-motion screw, obtain a perfect double contact.

Fig. 20.



It is a very good practice to set the horizontal limb of the theodolite to read 0° on the magnetic meridian by rotating the lower plate and clamping as it avoids any possible confusion in the azimuth of the referring mark (R.M.) and the magnetic variation which is the difference between the reading to the R.M. from magnetic north and the computed angle from true north.

This is done as follows :—Attached the compass to the theodolite on face left, unclamp the upper plate and set the A vernier to 0° and clamp the upper plate. Unclamp next the lower plate and revolve the instrument till the north end of the needle points north. Clamp the lower plate and release the upper plate and read the R.M.

An example is necessary to show how the declination of the sun is found from the N.A.

A set of observations was taken on the sun at a place keeping standard time 6 hours E of Greenwich on the 2nd June at 10 A.M., (mean of observed times). The following elements are extracted from the N.A. table II :—

Apparent declination 1st June N 21°. 59′. 03·8″ at G.M.N.

„ „ 2nd June N 22° 07′ 13·9″ „

From table 1. N.A. variation in 1 hour 1st June = $20{\cdot}90^{s}$ and 2nd June $19{\cdot}94^{s}$ 10 A.M., on the 2nd June Civil time (*see* a previous para. concerning change in G.M.T. for 1925 onwards) is 22 hours since noon on 1st June and since the time was for 6 hours E of Greenwich the corresponding Greenwich time was 16 hours on 1st June. By interpolation it will be found that the rate of variation at 16 hours is $20{\cdot}26^{s}$ which multiplied by 16 hours = 5′ 24·16″ and therefore the declination of sun at the time of observing was N 21°. 59′ − 03 8″ + 5′·24. 16″ = N 22° 04′ 28″.

∴ N.P.D. = 67°. 55. 32′.

When observing to a star for azimuth, as the object is very small and also as the declinations of stars change very gradually the recording of the date of observation is sufficient, and only the passage of the star across the intersection of the wires is required. The star should be allowed to intersect itself by manipulating either the horizontal or vertical screw for slow motion and allowing for the apparent inclined passage of the star.

In azimuth observations the lower plate will remain clamped throughout and as suggested, for preference, with 0° of A vernier on face "left" on the magnetic meridian.

The following example of observations and computations to a star *ex*-meridian will show how the field-book entry is kept, etc. As azimuths are for all practical purposes correct enough to a half minute the barometer and thermometer corrections are unnecessary. The footnotes on page. 85 are important.

FIELD-BOOK Latitude 29° 52′. *Azimuth ex*-meridian, dated *8th January*, 1914.

Face.	Object.	*Horizontal angles.*													*Vertical angles.*								Time and date
		A.			B.		Mean.			† Angle between R.M. and object.			A			B.		Mean			
		°	′	″	′	″	°	′	″	°	′	″	°	′	″	′	″	°	′	″	
L	R M.	23	59	20	59	40	23	59	30												
L	β Aurigæ E.	50	35	40	35	20	50	35	30				36	28	00	27	20				
R	"	230	46	40	47	00	50	46	40	333	10	05	[illegible]	28	40	28	40	37	54	02·5	7 P.M (approx.)
R	"	230	58	40	54	20	50	54	00				38	21	00	20	40				
L	"	51	01	00	00	40	51	00	50				39	19	20	18	40				
R	R.M.	203	59	00	59	20	23	59	10												
L	R.M.	23	59	20	59	40	23	59	30												
L	α Pegasi W	254	58	40	59	00	254	58	50				48	14	20	14	20				
R	"	75	58	40	53	40	255	53	40	127	46	20	47	06	20	06	20	46	40	10	7·15 P.M. (approx.)
R	"	76	36	20	36	40	256	36	30				46	12	20	12	40				
L	"	257	24	40	25	20	257	25	00				45	07	20	07	40				
R	R.M.	203	59	40	00	00	23	59	50												

COMPUTATION. *8th January, 1914.*

	°	′	″	°	′	″
Object E. or W. ...	*β* Aurigæ E.			*α* Pegasi W.		
Mean Zenith Distance ...	52	05	57	43	19	50
+ Refraction − Parallax ...	+	01	15	+	00	55
Corrected Z. D. $= p$	52	07	12	43	20	45
Co'atitude $- o$	60	08	00	60	08	00
N. P. Distance $= z$	45	03	25	75	15	26
Sum $= 2s$	157	18	37	178	44	11
s	78	39	19	89	22	06
$s - z$	33	35	54	14	06	40
$s - o$	18	31	19	29	14	06
$s - p$	26	32	07	46	01	21
Log cosec s ...	0	008	5695	0	000	0264
,, ,, $(s - z)$ ..	0	256	9865	0	612	9608
,, sin $(s - o)$ ...	$\bar{1}$	501	9733	$\bar{1}$	688	7693
,, ,, $(s - p)$ ...	$\bar{1}$	650	0633	$\bar{1}$	857	0987
Sum = Log tan² ½*A* ...	$\bar{1}$	417	5926	0	158	8552
,, tan ½*A* ...	$\bar{1}$	708	7963	0	079	4276
½*A* ...	27	05	14	50	12	38
*A** ...	54	10	28	100	25	16
Angle R. M. and Star† ...	333	10	05	127	46	20
Azimuth of R.M. from N. ...	27	20	33	27	21	04

* When the Star is $\frac{\text{east}}{\text{west}}$ of the meridian the azimuth of the Star is to be $\frac{\text{added to}}{\text{subtracted from}}$ the angle R. M. and Star; if the result is *negative*, subtract it from 360°; and if positive but greater than 360° deduct 360° from it. To the result thus obtained convergency is to be $\frac{\text{added to}}{\text{subtracted from}}$ azimuth observed $\frac{\text{west}}{\text{east}}$ of origin. For convergency correction *see* para. 131 Part I. (Convergency is roughly 30″ per mile in Lat.

† Always *subtract* the reading of the Star from that of the R. M. to get the angle R. M. and Star.

If the above azimuth had been taken 2 miles E. of the origin the bearing to be used in ordinary traverse computation would have been $27° 21' - 1' = 27° 20'$. Magnetic Variation $3\frac{1}{2}°$ E (approx.)

68. **Azimuth to a circumpolar star at elongation.***– A circumpolar star is one whose N. P. D. is less than the latitude of the place, or in other words, whose declination is greater than the colatitude of the place. It can thus easily be conceived that a circumpolar star never sets or goes below the horizon of the place at which the observation is taken. Polaris is the star usually observed and a few facts concerning Polaris would not be out of place here. If the times of W. or E. elongation of Polaris is not suitable any other circumpolar star will give just as good results. Polaris is known as *α* Ursæ Minoris or the bright star of the "Little Bear" (in America this constellation is known as the "Little Dipper)." The "Great Bear" or Ursæ Majoris has two stars pointing direct to Polaris. The "Great Bear" is known sometimes as the "Great Dipper" and sometimes as the "Plough." The pointers are at the end of the dipper opposite to the handle. The last star but one in Ursæ Majoris is ζ Ursæ Majoris or "Mizar." When Polaris is vertically over Mizar then it is nearly on the meridian.

The polar distance of Polaris is now about 1° 07′, this distance will decrease at the rate of $\frac{1}{3}$ minute a year until the star is about 30 minutes from the pole when it will begin to increase.

When we say the N. P. D. of Polaris is 1° 07′ its N. P. D. will be 1° 07′ at the equator, that is, that the angle of 1° 07′ from the meridian will only hold good on the equator or at latitude 0° when the zenith and celestial equator coincide. Directly the observer moves north his zenith approaches the pole, and although the N. P. D. of the star remains constant, at elongation the angle at the zenith between the star and pole increases. At N. latitude 40° it would equal 1° 22′ 13″ or sin star's bearing $= \frac{\sin NPD}{\cos lat}$ since $\frac{\sin PZO}{\sin ZOP} = \frac{\sin PO}{\sin ZP}$ and $ZOP = 90°$ and $ZP = 90° - \lambda$. For this reason the correction for latitude enters into the computations.

By consulting Napier's rules of circular parts (para. 61) when the angle at O, the object, is 90° we obtain the following:—

$$\sin z = \cos\left(\frac{\pi}{2} - Z\right)\cos\left(\frac{\pi}{2} - o\right) = \sin Z \sin o.$$

$$\therefore \sin Z = \sin \text{azimuth angle} = \frac{\sin NPD}{\sin colat.} = \frac{\sin NPD}{\cos lat.} \quad \text{..............(i).}$$

*A star is said to be at elongation when the angle at the star between the plane of the declination circle passing through the star and the plane of the vertical circle passing through the star is 90°.

again $\sin\left(\frac{\pi}{2} - o\right) = \cos p \cos z$.

$\therefore \cos p = \text{sin altitude} = \frac{\cos o}{\cos z} = \frac{\text{sin latitude}}{\cos \text{ NPD}}$ ....(ii).

and again $\sin\left(\frac{\pi}{2} - \text{P}\right) = \cos \text{P} = \cos h.\ a. = \tan\left(\frac{\pi}{2} - o\right) \tan z$

$= \text{tan latitude} \times \tan \text{N.P.D}$.................(iii).

and thus we find the angle, time and altitude of any circumpolar star at eastern or western elongation. The altitude it must be remembered is the true altitude uncorrected for refraction.

Example on the 11th November 1915. The following elements were given :—

	η Draconis (W)	ζ Draconis (W)
RA	$16^h.\ 22^m.\ 49^s$	$17^h.\ 08^m.\ 30^s$
N.P.D.	$28^\circ.\ 17'.\ 45''$	$24^\circ.\ 10'.\ 51''$
S.T. at L.M.N.	$15^h.\ 17^m.\ 20^s$	$15^h.\ 17^m.\ 20^s$
Lat	$29^\circ.\ 52'.\ 00''$	$29^\circ.\ 52'.\ 00''$

Therefore considering the formulæ for Cos P, sin Z and sin alt. the following computation is necessary.

tan λ	$\bar{1}$·7591022	$\bar{1}$.7591022
tan N.P.D.	$\bar{1}$·7310653	$\bar{1}$·6522616
Log Cos P	$\bar{1}$·4901675	$\bar{1}$·4113638
	h ′ ″	h ′ ″
+ h.a.	4·47·58	+ 5·00·14
R.A.	16·22·49	17·08·30
L.S.T. of obs :	21·10·47	22·08·44
ST @ LMN	15·17·20	15·17·20
Sid Int. from LMN	5·53·27	6·51·24
Retard :	58	1·07
Mean Time (Local)	5·52·29	6·50·17
Correction for Stand : T	18·24	18·24
	h ′ ″	h ′ ″
Chronometer time	6·10·53	7·08·41
Sin NPD	$\bar{1}$·6758005	$\bar{1}$·6123789
Cos λ	$\bar{1}$·9381126	$\bar{1}$·9381126
Log Sin Z	$\bar{1}$·7376879	$\bar{1}$·6742663
Azimuth angle (Z)	33°·03′·08″	28°·11′·14″
Sin λ	$\bar{1}$·6972148	$\bar{1}$·6972148
Cos NPD	$\bar{1}$·9447352	$\bar{1}$·9601173
Log Sin alt	$\bar{1}$·7524796	$\bar{1}$·7370975
alt	34°·26′·29″	33°·05′·05″
refraction	+ 1·23	+ 1·28
app. altitude at elongation	34°·27′·52″	33°·06′·33″

The above shows that the time of Elongation of one star by the clock keeping standard time was 6 h. 10 m. and 53 s. and of the other star was 7 h. 08 m. 41 s.

To take the observation set up and level a theodolite very accurately over a mark (generally a traverse station) and as advised, in a previous para. to avoid mistakes, put on the magnetic compass, set the two plates on face left A vernier to 0,° open or release lower clamp and rotate instrument till the needle of the compass shows 0.° Clamp the lower plate. The 0° line of the instrument is now with reference to the magnetic meridian.

Set up a lamp on the R.M. which will be either on the back or forward traverse station as a referring mark (R.M) and read the angle to it on both faces. Let such a mean angle be 56° 52′ 20.″ This should bring you up to a few minutes before the time of elongation, and in case your clock is not very correct it would be as well to leave a margin of a few minutes, the clock time being corrected and an allowance made for local time as against standard time. For η Draconis set the vertical arc to 34° 26′ 29″, and if you do not quite know the star set the horizontal vernier to read 360°—33° 08′ roughly. The star will be recognisable and as the altitude has not been augmented for refraction the star has still to rise higher in the telescope and therefore has not yet reached its elongation. Watch it now very carefully and immediately it is seen to be rising vertically along the wire (falling in the telescope for eastern elongation) clamp and read the *horizontal* plate, and if you have time change face and read again to disperse collimation error. Polaris for 10 minutes or so of time on either side of elongation does not change more than 10 secs. in arc and therefore with Polaris there is sufficient time to take it on both faces. In case both faces are not taken, the reading to the star will be referred to that of the R.M. on the same face only. Having now obtained the mean azimuth angle on the theodolite we can find the line of true north by subtracting the computed value from the observed value.

Correct now the magnetic bearing of the R.M. to obtain azimuth of R.M.

Note that the time and altitude, except as a rough guide for the position of the star at elongation, do not enter further into the calculations. The R.M. may be observed after the star. It is neither advisable nor possible to leave an instrument standing till daylight to observe to a peg or any defined mark, and a lamp on the R.M. provided the R.M. is not too close is sufficiently accurate as a mark for all traverse purposes in which an azimuth to ½ minute in correctness is deemed necessary. Convergency correction to be applied as in para. 131, Part I.

69. **To find the meridian by Polaris.**—This method is given in Taylor's "Handbook for Surveyors", and is worthy of note, as no nautical almanac is required, and it is not every engineer who has an almanac, and with the following tables, a rough knowledge of the observer's latitude and a fairly good idea of the *local* time of observation, a correct meridian is possible.

Table I. gives the epochs or times of equal date in *April* when the Mean Sun and Polaris are on a meridian together that is when the apparent R. A. of Polaris and the R. A. of the Mean Sun agree.

Year.	Epoch.	Year.	Epoch.
1924	15·0	1930	16·4
1925	15·7	1931	16·6
1926	16·1	1932	16·0
1927	16·4	1933	16·4
1928	15·7	1934	16·8
1929	16·1	1935	17·1

In 1924 with Epoch 15·0 showed that the mean sun and Polaris were on a meridian together on the 15th April at 12 midnight (14th and 15th). The next day the sun will reach the meridian nearly 4 mins. later than Polaris, and thus the hour angle of the star will be more than that of the sun by 3·94 minutes multiplied by the number of days after the epoch to which must be added the angle between the sun and Polaris on that day and if we call this hour angle t, from Table II., given below, we obtain an angle which multiplied by the azimuth co-efficient in Table III. gives us the correct azimuth.

TABLE II. TABLE III.

Hours (t).	Angle (a).	Hours (t).	Lat.	1920.	1930.	1935.
0	0′	24				
1	25′	23				
2	49′	22	20°	·75	·72	·69
3	69′	21				
4	84′	20				
5	93′	19	30°	·81	·77	·73
6	96′	18				
7	92′	17				
8	82′	16	40°	·91	·87	·82
9	67′	15				
10	47′	14				
11	24′	13	50°	1·09	1·04	·99
12	0′	12				

Example.—On the 7th January 1925 at 6-40 p.m., L. M. T. the angle between Polaris and a Referring Mark was found to be 121° – 54' — 00". What was the azimuth of the R. M. if the latitude of the place was 29° 52' N.?

Here the Epoch 15·0 for 1924 must be considered and the number of days lapsed since 14th—15th midnight of April 1924 to 6 hours 40 minutes p.m. L.M.T. on the 6th January 1925. To simplify matters it is better to count in full numbers of days from the 1st April of any one year to the full date of observation and subtract the value given in the Epoch. In this case we have the number of days as 30 + 31 + 30 + 31 + 31 + 30 + 31 + 30 + 31 + 6·78 = 281·78 – 15·0 days = 266·78 days and thus the hour angle between the mean sun and apparent R. A. of Polaris at the time of observation, since the gain is 3·94 minutes per diem, is 266·78 days × 3·94 = 17·5 hours.

We next want to know the position of Polaris with respect to the *meridian* and the position of the meridian is known since the sun was 6·66 hours past it and thus the angle of Polaris with respect to the meridian or t = 17·5 + 6·66 = 24·16 hours = 0·16 hours past the meridian (in the first quadrant).

From Table II. we obtain 25' × ·16 = 4 mins.

From Table III. for latitude 29° 52' and year 1925 we obtain ·79 (by interpolation) as a multiplier.

Therefore Azimuth of Polaris was 4 × ·79 = 3·08 mins west = 3' 04·8".

Therefore Azimuth of R M = 121°·54'·00" – 3'·05" = 121°50'55".

Example.—What was the azimuth of Polaris at 6 hours 28 minutes L.M.T. on the 12th February 1926?

Days elapsed since 1st April 1925 = 30 + 31 + 30 + 31 + 31 + 30 + 31 + 30 + 31 + 31 + 11·78 = 317·78 days and epoch for 1925 = 15·7. Therefore the number of days elapsed since Polaris and the sun were on a meridian together = 317·78 – 15·7 = 302·08; which multiply by 3·94 giving as a result 19 hours 50 minutes.

To find the position of Polaris with respect to the meridian, 6 hours 28 minutes must be added to 19 hours 50 minutes which equals 2 hours 18 minutes. Thus the star is in the 1st quadrant (W. of North) and t = 2 hours 18 minutes. Interpolating in Table II. we obtain 8 = 55' 00" and reducing by Table III. the true azimuth of Polaris W. of North is 55' × ·79 = 43 minutes 27 seconds West.

70. Observations for Time ex-meridian to a fixed star or the sun.—It is necessary, to get the best results by observations to a fixed star, to select one on the *prime vertical* when the apparent

movement of a star is greater and therefore gives the best time results.* If the latitude of the observer is 30° N. then stars in the N.A. with a declination N. of 30° will be on the prime vertical.

Set up the theodolite and level it very exactly, specially the upper level or vertical arc level. If this level is fixed on the telescope it is necessary to first set the vertical arc to 0°. If the theodolite is not fitted with a reflecting cap take a band of white paper about 2½ inches wide, fit it over the object glass end of the telescope with a pin, and tear off a small quantity overlapping the object glass and leaving, so to speak, a small tongue which is then bent inwards at an angle of 45° or so. The light of a lamp or a small lantern thrown on to this bent tongue will be reflected down the telescope and will illumine the wires of the diaphragm and the light can be regulated either by altering the angle of the bent piece or by holding the lamp closer or further away from the paper. The correct illumination will have been obtained when the wires and star are equally visible. Too much reflected light is a mistake. To get the star in the view of the telescope arrange that the lanterns, etc., near by are held away from the theodolite, then sight along the top of the telescope to the star, and the star should be in the field of view of the telescope. Some theodolites of large pattern are fitted with gun sights, but a little practice with the ordinary theodolite will make this preliminary work quite easy, the observer remembering that the direction of the motion of the star is inverted in the telescope, and that a *bright star to the naked eye will appear a bright object in the telescope.*

The observer should set his telescope on any bright star, focus it to a point so to speak, disperse parallax, and he is ready then to commence work. The star he selects † is next brought in view in the telescope, his vertical level is examined and the light is reflected on to his diaphragm. He next sets the star as close to the vertical wire as possible if the star is in the east above the horizontal wire, and if in the west below it.‡ In the ordinary theodolite it often happens that the lenses are only optically correct near the centre and observations should be made as close to the field of cross hair as possible. The flare seen through the telescope means that the lens is not set square and to be true the star should focus to a steady point so to speak. Having finally ascertained that the levels are correct he calls "ready" to the recorder and time-keeper, and if there is no recorder he should test his counting of seconds by his watch and try and obtain as near as possible the correct

* When the mean of E and W stars for Time or the mean of N and S stars for latitude is accepted there is no need to change face R. G. S. Journal June 1917.

† Stars of the first magnitude or very bright stars do not give such good results as those of lesser magnitudes.

‡ In Time observations he should not observe the star at the actual intersection of the cross wires as this is where the glass is often splintered by the engravers tool.

beat; he goes on counting till the star crosses the *horizontal* wire and making a mental note of the intersection, continues counting and examines his watch to see what error has accrued in the interval; if he is not satisfied he should do it again, but with a little practice and experience a few seconds need only lapse for the whole operation of counting, etc., so that the "beat" error is a negligible quantity.*

If there is a recorder he, on the caution "ready," starts counting the seconds and calls them loudly, and the observer at the crossing calls the second he wishes registered. It is not a good practice to say "now" "up", etc. The recorder enters the seconds called and makes a careful note of the minutes (the watch or clock should be set before observation so that the full minute of the minute hand is reached when the second hand shows 0 seconds). Mistakes are usually made in the minutes when recording just as in levelling the feet are often recorded wrongly because the leveller is too intent on getting the decimals correct. He next changes face and goes through the same operation in exactly the same order, the recorder noting that if the star is in the east the next values given him will be greater in altitude and *vice versâ* if the star is in the west. It is not advisable to take observations to a star of lower elevation than 25^0 owing to the refraction correction increasing and being less reliable.

The observer should next take a star in the west if his first star was in the east. This balances the observation on each side of the meridian and also eliminates the personal error of taking a star sometimes slightly above or slightly below the horizontal wire. Since the conditions of incorrect intersection are, so to speak, equal and opposite the mean of the times on east and west disperses this personal error. Thus, the altitude of a certain N.A. star has been observed and connected to the time of a clock or watch on a certain date, and it has already been explained how, if latitude is known also the declination of a star and if altitude (corrected for refraction) is found, the spherical triangle can be solved when the hour angle is found and, *vide* para. 63, the error of the watch can be calculated.

The following steps in observing are recommended:—

Adjust theodolite, sight star, focus and regulate the lighting, call "ready," intersect star, call second of intersection, read vertical arc verniers (object end first), change face, etc.

In *time* observations there is no point in centering an instrument as a fairly large error in the assumed longitude or a small error in the assumed latitude will not give an appreciable error in the time result.

* If the sentence "a thousand and one" is repeated the exact beat of one second of time will occur on the word "one."

The latitude and longitude are found on the standard maps of the locality usually to a scale of one inch = one mile, and of course there is no point in clamping or otherwise using the horizontal plate of the instrument.

When the sun is the object observed the best method is to take the time when a limb touches the horizontal wire and without altering the *vertical* angle to take the time when the opposite limb leaves the wire. The mean of the two times and the one vertical angle will give on one *face* the apparent altitude and time of the crossing of the sun's centre. Another way is to take only one limb and allow for the correction for semi-diameter. In any case on account of collimation error observations should be on both *faces*.

Example 1.—Observations for Time with T. and S. 6-inch theodolite No. 1170. Latitude 18° 30′ 23″. Longitude 73° 53′ 06 E. Dated 17-5-1907 Chronometer keeping standard time for longitude 82° 30′ E.

Face	Object E. or W.	Vertical angles. A.			B.		Mean			General Mean.			Time.			Mean of time.		
		°	′	″	′	″	°	′	″	°	′	″	h.	m	s	h.	m.	s.
R.	Procyon, W. ...	24	48	40	48	40	24	48	40				8	59	32			
L.	,, ...	24	00	40	00	40	24	00	40	23	39	03	9	02	53	9	04	16
L.	,, ...	23	09	40	09	20	23	09	30				9	06	07			
R.	,, ...	22	37	00	37	40	22	37	20				9	08	31			
L.	α Ophiuchi E. ...	24	31	40	32	20	24	32	00				10	00	27			
R.	,, ,, ...	25	25	40	26	00	25	25	50	25	41	39	10	04	01	10	05	17
R.	,, ,, ...	25	55	40	55	50	25	55	45				10	06	08			
L.	,, ,, ...	26	53	00	53	00	26	53	00				10	10	30			

Example 2.—Observations for Time with T. and S. 6-inch theodolite taken at Roorkee on October 30th 1923. Latitude 29° 52′ 00″. Longitude 77° 53′ E. Chronometer keeping standard time or 18′-24·5 secs. fast of L.M.T.

L.	α Ophiuchi W. ...	43	20	20	21	20	43	20	50				6	26	11			
R.	,, ...	317	47	40	46	50	42	12	45	41	55	11	6	31	32	6	32	52
R.	,, ...	138	35	50	37	10	41	23	30				6	35	18			
L.	,, ...	40	43	20	44	00	40	43	40				6	38	27			
L.	α Andromedæ E ...	49	00	40	02	20	49	01	30				6	42	07			
L.	,, ...	49	30	20	30	40	49	30	30	49	54	30	6	44	23	6	46	15
R.	,, ...	129	48	30	48	30	50	11	30				6	47	35			
R	,, ...	129	05	50	05	10	50	54	30				6	50	55			

Computation of Time.

Example 1. *Example* 2.

Station	Poona.			Poona.			Roorkee.			Roorkee.		
Date Ast.	17	5	07	17	5	07	30	10	23	30	10	23
	°	′	″	°	′	″	°	′	″	°	′	″
Longitude	73	53	06	73	53	06	77	53	E	77	53	E
Object E. or W.	Procyon (W)			α Ophiuchi (E).			α Ophiuchi (W)			α Andromedæ (E)		
Mean obsd. Z.D.	66	20	57	·64	18	21	48	04	49	40	05	30
Refraction − Parallax	+	2	13	+	2	00	...	1	05	...	...	49
Corrected Z D. $= p$	66	23	10	64	20	21	48	05	54	40	06	19
N. P. D. $= z$	84	32	20	77	22	25	77	22	50	61	19	40
Colatitude $= o$	71	29	37	71	29	37	60	08	00	60	08	00
Sum $= 2s$	222	25	07	213	12	23	185	36	44	161	33	59
Half " $= s$	111	12	34	106	36	12	92	48	22	22	46	59
$s - p$	44	49	24	42	15	51	44	42	28	40	40	40
$s - p$	26	40	14	29	13	47	15	25	32	19	27	19
$s - o$	39	42	57	35	06	35	32	40	22	20	38	59
Log cosec s	0	030	4610	0	018	4958	0	000	5211	0	005	6437
" " $(s - p)$	0	151	8581	0	172	2755	0	152	7413	0	185	8828
" sin $(s - z)$	$\bar{1}$	652	1107	$\bar{1}$	688	6978	9	424	8590	9	522	5368
" " $(s - o)$	$\bar{1}$	805	4875	$\bar{1}$	759	7766	9	732	2654	9	547	3486
Sum = Log $\tan^2 \frac{1}{2} t$	$\bar{1}$	639	9173	$\bar{1}$	639	2457	19	310	3868	19	261	4119
" $\tan \frac{1}{2} t$	$\bar{1}$	819	9587	$\bar{1}$	819	6229	9	655	1934	9	630	7059
(arc) $\frac{1}{2} t$	33	27	00·0	33	25	46·5	24	19	33	23	08	09
(time) $\frac{1}{2} t$	2	13	48·0	2	13	43·1	1	37	18·2	1	32	32·6
	h.	*m.*	*s.*	*h.*	*m.*	*s.*	*h.*	*m.*	*s.*	*h.*	*m.*	*s.*
(*h. a.*) or **t* in time	4	27	36	− 4	27	26·2	+ 3	14	36·4	− 3	05	05·2
✱'s R.A. or ☉'s Eq of T.	7	34	25·4	17	30	38·3	+17	31	22·5	+ 0	04	27·8
For stars only. Local S.T. of obsn.	12	02	01·4	13	03	12·1	20	45	58·9	20	59	22·6
For stars only. S.T. at Local M.N.	− 3	~~32~~5	19·9	− 3	35	19·9	14	30	15·6	14	30	15·6
For stars only. S. Interval from "	8	26	41·5	9	27	52·2	6	15	43·3	6	29	07·0
For stars only. Retardation	−	01	23	−	01	33·0		1	01·4		1	03·6
True M. T. of obsn.	8	25	18·5	9	26	19·2	6	14	41·9	6	28	03·4
Chr. Time	9	04	16	10	05	17·0	6	32	52	6	46	15
Chr. Error	+	38	57·5	+	38	57·8		18	10·1		18	11·6

Example 2.—Altitude of the sun's upper and lower limbs observed. Long. 77°·53′ Thermometer, 70° F.; Barometer, 28·85 inches. Civil date 5-1-14 A.M.

The observation was conducted as follows:—As the sun was rising the time was noted when the upper limb of the sun crossed the horizontal

* *t* is ∓ when star is $\frac{\text{East}}{\text{West}}$ of Meridian and Apparent Time is 12h − *t* if sun is East of Meridian.

wire of a transit instrument, and again noted when the lower limb crossed. The altitude was then read :—This, therefore, was the altitude of the sun's centre at the mean of the two observed times. The altitudes here given are a mean between two observations ; and the time, the mean of four observations. The working out of this example is left as an exercise.

1st set sun's observed altitude of centre } = $34^0\ 28'\ 47{\cdot}5''$, time $11h.\ 15m.\ 51{\cdot}3s.$

2nd „ „ „ „ = $35°\ 26'\ 15{\cdot}0''$, „ $11h.\ 29m.\ 23{\cdot}5s.$

Declination of sun for date and hour ... ... $\lambda 29°{\cdot}52'$

watch or clock was keeping standard time for Longitude $82^0\ 30'$ E., therefore $18m.\ 24{\cdot}5s.$ must be subtracted *from the above* times to find the true watch error.

t in time represents the true angle east of noon of the apparent time or sun time was 12 hours minus 1 hr. 14 mins. 29·4 secs. A.M. = 10 hrs. 45 mins. 30·6s. A.M. on the 5th January, which, in astronomical time, is 22 hrs. 45 mins. 30·6 secs. on the 4th January at the locality at which the observation was made, or roughly $22\frac{3}{4}$ hrs. – $5\frac{1}{2}$ hrs. (difference in longitude E. of Greenwich = $17\frac{1}{4}$ hours at Greenwich on the 4th January. The equation of time on the 4th January at Greenwich is 4 mins. 51·5 secs. *to be added* ; variation per hour 1·145 secs. increasing ; hence the result 5 mins. 11·7 secs. which must be added to apparent time to obtain mean time.

For observations to celestial objects it is important to remember the following :—

(1). Readings are read on both faces of the instrument to correct instrumental errors.

(2). Two separate observations to correct mistakes

(3). An east and a west star to correct personal error of intersection on the wire.

(4). Stars of equal azimuth are selected to correct or counteract any error in the assumed latitude supposing the correct latitude is not known. اردلب

(5). Stars of equal altitudes to correct for refraction.

71. **Latitude***.—When Polaris is at its upper transit and its altitude is corrected for refraction then by subtracting its N.P.D. from the resulting altitude the latitude of the place is found, and similarly adding its N.P.D. to the corrected altitude of its lower transit the latitude of the place is found, and hence we also obtain the rule that the latitude of a place

* Local attraction may cause the Astronomical latitude to differ from the Geodetic by several seconds owing to the attraction of mass on the pendulum or plumbob. This difference cannot be eliminated by any process of star observations whatsoever.

is the mean of the corrected altitudes of a circumpolar star observed at its upper and lower culminations or transits.

To observe to Polaris *ex*-meridian and then compute the results there are two methods—one from formula and the other from the tables given in the Nautical Almanac. They will both be treated of here.

By formula—

It has been given elsewhere that $\cos p = \cos o \cos z + \sin z \sin o \cos P$......2 : wherein P is the hour angle or t and if h = altitude, ϕ latitude and p' = NPD = z we have $\sin h = \sin \phi \cos p' + \cos \phi \sin p' \cos t$. ....2 (*a*) ; t the hour angle and h the altitude are derived by observation and ϕ is the required latitude. Now the N.P.D. or p being small (at present less than $1^0\ 10'$) we may develop ϕ in a series of ascending powers of p and then employ as many terms as we need to attain any given degree of precision. The altitude cannot differ from the latitude by more than p'; if then we put $\phi = h - x$; x will be a small correction of the same order of magnitude as p, and we have by Taylor's and Maclaurin's theorems—

$$\text{Sin}\ \phi = \sin (h - x) = \sin h - x \cos h - \tfrac{1}{2} x^2 \sin h + \tfrac{1}{6} x^3 \cos h + \&\text{c.}$$

$$\text{Cos}\ \phi = \cos (h - x) = \cos h + x \sin h - \tfrac{1}{2} x^2 \cos h - \tfrac{1}{6} x^3 \sin h + \&\text{c.}$$

$$\text{Sin}\ p' = p' - \tfrac{1}{6} p'^3 + \&\text{c., and } \cos p' = 1 - \tfrac{1}{2} p'^2 + \&\text{c.}$$

These values substituted in equation 2 (*a*) give us—

$$\text{Sin}\ h = \left[\sin h - x \cos h - \frac{x^2}{2} \sin h + \&\text{c.}\right]\left[1 - \frac{p'^2}{2} + \&\text{c.}\right] +$$

$$\left[\cos h + x \sin h - \frac{x^2}{2} \cos h - \frac{x^3}{6} \sin h + \&\text{c.}\right]\left[p' - \frac{p'^3}{6} + \&\text{c.}\right] \cos t$$

$$= \sin h - x \cos h - \frac{x^2}{2} \sin h - \frac{p'^2}{2} \sin h + p' \cos h \cos t$$

$$+ xp' \sin h \cos t + \&\text{c., or } \sin h = \sin h - x \cos h + p' \cos t$$

$$\cos h - \tfrac{1}{2} (x^2 - 2x p' \cos t + p'^2) \sin h + \&\text{c.,}$$

then $x \cos h = p' \cos t \cos h - \frac{1}{2} (x^2 - 2xp' \cos t + p'^2) \sin h + \&\text{c.}$.........

and $x = p' \cos t - \frac{1}{2} (x^2 - 2xp' \cos t + p'^2) \tan h + \&\text{c.}$,............(*a*),

for the first approximation we take $x = p' \cos t$ and substituting this in equation (*a*) neglecting the third powers of p' and x we obtain a second approximation thus—

$$x = p' \cos t - \tfrac{1}{2} (p'^2 \cos^2 t - 2 p'^2 \cos^2 t + p'^2) \tan h + \ldots\ldots\ldots\ldots$$

$$= p' \cos t - \tfrac{1}{2} (- p'^2 \cos^2 t + p'^2) \tan h \ldots\ldots\ldots\ldots\ldots\ldots$$

$$= p' \cos t + \tfrac{1}{2} p'^2 (\cos^2 t - 1) \tan h = p' \cos t + \tfrac{1}{2} p'^2 (- \sin^2 t) \tan h$$

$$= p' \cos t - \tfrac{1}{2} p'^2 \sin^2 t \tan h,$$

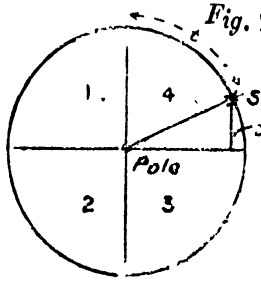
and there is no need to go further with the expression to find latitude to within 1" and since the angle must be in circular measure,

$$x = p' \cos t - \tfrac{1}{2} p'^2 \sin 1'' \sin^2 t \tan h$$

$$\text{or } \phi = h - p' \cos t + \tfrac{1}{2} p'^2 \sin 1'' \sin^2 t \tan h.$$

If we take the first part of the equation without the small correction we have $x = p' \cos t$ or $\cos t = \frac{x}{p'}$ and

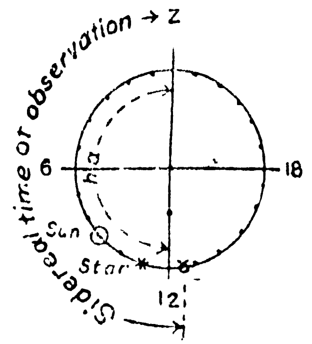
Fig. 21.



vide diagram $\sin(90\text{-}t) = \frac{x}{p'} = \frac{x}{\text{N.P.D.}}$ and to find the altitude of the pole in quadrants 1 and 4 the formula will be $h - x$ and in quadrants 2 and 3 $= h + x$. From this the latitude is found plus the small correction $\frac{1}{2} p'^2 \sin 1'' \sin^2 t \tan h$.

As before explained t is the hour angle in arc, so it is first necessary to find what the $h.a.$ in time is before the computation can be proceeded with and to find the $h.a.$ in time we must first find the local sidereal time of observation and subtract from it the R.A. (*see* diagram for rough approximation).

Fig. 22.



If t in arc is in the 2nd or 3rd quadrant then $p' \cos t$ will have the opposite sign but $\frac{1}{2} p'^2 \sin 1'' \sin^2 t \tan h$ is always additive.

Latitude from Polaris.

Field Book. Bar. 28·5. Therm. 76,° dated 17-5-1907.

Face.	Object E or W.	Vertical angles A °	A ′	A ″	B ′	B ″	Mean °	Mean ′	Mean ″	General Mean °	General Mean ′	General Mean ″	Time h.	Time m.	Time s.	Mean of time.* h.	m.	s.
R	Polaris	17	22	20	21	50	17	22	05				9	13	29			
L	(α Ursæ Minoris)	17	25	40	25	50	17	25	45	17	23	39	9	21	51	9	22	34
L		17	25	20	25	30	17	25	25				9	25	35			
R		17	21	20	21	20	17	21	20				9	29	21			
R	γ Centauri	23	02	40	03	00	23	02	50				9	33	51			
L		23	06	20	07	00	23	06	40				9	37	12			
L		23	07	00	06	40	23	06	50				9	40	51			
R		23	03	00	02	40	23	02	50	23	02	50	9	45	07	9	46	00
R		23	01	40	02	00	23	01	50				9	48	09			
L		23	02	20	03	20	23	03	00				9	50	54			
L		23	01	00	01	20	23	01	10				9	53	54			
R		22	57	50	57	10	22	57	30				9	56	05			

* Compare example I.—Computation for Time—which observations were taken before and after the above to obtain watch or clock error.

Computation for latitude by the foregoing formula.

		h. m. s.
Mean of watch times	=	9 22 34
Chronometer error (*see* example I., page 94) (for L.M.T. not Stand : Time).	=	− 0 38 58
True local mean time of observation	=	8 43 36
Acceleration	=	1, 26
Sidereal Interval from L.M.N.	=	8 45 02
Sidereal time at L.M.N.	=	3 35 19·9
*Sidereal time of observation (that is the time since Aries was on the meridian).	=	12 20 21 9

See Fig. 22.

Latitude $= h - p' \cos t.$
$+ \frac{1}{2} p'^2 \sin 1'' \sin^2 t \tan h.$

		h. m. s.
Sidereal time of observation	=	12 20 21·9*
star's R.A.	=	1 24 57·98
hour angle in time (*see* diagram Fig. 22)	=	10 55 23·92

∴ hour angle in arc $= 163^\circ\ 50'\ \ 59''$ (2nd quadrant)

and p' = N.P.D. = $1^\circ\ 11'\ 35{\cdot}23''$ for Polaris on 17-5-1907
$= 4295{\cdot}23$ secs.

∴ $\log p' = 3{\cdot}6329864$	$\log p'^2 = 7{\cdot}2657$
$\log \cos t = \bar{1}{\cdot}9825134$	$\log \sin^2 t = \bar{2}{\cdot}8886$
	$\log \tan h = \bar{1}{\cdot}4947$ $(17^\circ\ 20'\ 51{\cdot}86'')$.
$3{\cdot}6154998$	$\log \frac{1}{2} \sin 1'' = \bar{6}{\cdot}3845$ (log sin 1″ − log 2).
$= 4125{\cdot}71''$	$0{\cdot}0335$
$= 1^\circ\ 08'\ 45{\cdot}71''$	$1{\cdot}08''$

∴ latitude $= 17^\circ\ 20'\ 51{\cdot}86'' + 1^\circ\ 08'\ 45{\cdot}71'' + 1{\cdot}08''$
$= 18^\circ\ 29'\ 38{\cdot}65''$.

Latitude by an altitude of a star observed in any position can be found very simply if a Time observation has been taken. The calculations are based on the formula as before but N. P. D. is substituted for δ and we get

$$\operatorname{Sin} h = \sin \phi \sin \delta + \cos \phi \cos \delta \cos t.$$

Where ϕ is the only unknown quantity.

* Sidereal time of observation = S. Int. from L.M.N. + ST at L.M.N.
∴ R.A. ± h. a = S.T. of observation ∴ h. a = S.T. of observation—R.A. (para. 63).

By introducing two auxiliaries d and D determined by the equations.

$$d \sin D = \sin \delta \quad \ldots\ldots (a)$$
$$d \cos D = \cos \delta \cos t \quad \ldots\ldots (a')$$

and by substituting the value of d from (a) the equation becomes.

$$\cos(\phi - D) = \sin h \sin D \operatorname{cosec} \delta.$$

$$\text{also } \frac{(a)}{(a')} = \tan D = \tan \delta \sec t.$$

This will give two values for latitude but in computing Example 2 Form K a value of latitude 29° 52′ has been accepted as near enough and the deduced value from the formula above will show which one of the two is obvious.

From example 2 Form K we obtain the following :—

	a Ophiuchi.	*a Andromedæ.*
declination = δ =	12° 37′ 10″	28° 40′ 20″
corrected altitude = h =	41° 54′ 06″	49° 54′ 41″
t (arc) =	48° 39′ 06″	46° 16′ 18″
log tan δ =	9·3500208	9 7378714
log cos t =	9·8199617	9·8396288
log tan D =	9·5300591	9·8982426
∴ D =	18° 43′ 16″	33° 20′ 54″
log sin h =	9·8246817	9·8835831
log sin D =	9·5064536	9·7927004
log cosec δ =	0·6605994	0·3189414
log cos (ϕ — D) =	9·9917347	9 9952249
ϕ — D =	11° 08′ 34″	8° 28′ 57″
∴ ϕ =	29° 51′ 50″	29° 52′ 03″

72. **By circum-meridian altitude**.—For various reasons, the mean of several observations on a star, when they are taken closely following each other, and at about equal intervals of time, is more to be depended upon than any one of the individual observations. As only one observation can be taken when the star is crossing the meridian and vertical collimation supervenes it is more convenient to take a series of observations when the star or heavenly body is near the meridian, and reduce each observation to the meridian. This method is susceptible of very great accuracy, and has the advantage of being able to be performed either by a theodolite or a sextent ; and, besides, the exact position of the meridian is not required to be known, consequently the observations necessary to lay down the meridian can be dispensed with.*

* For this observation, the error of the clock must be accurately known, so as to allow of the mean time of apparent noon being determined.

The formula for the reduction to the meridian is x (in secs.) $= \frac{2 \sin^2 \frac{1}{2} t}{\sin 1''}$ cos (approx. lat.) sin N.P.D. cosec (approx.) Z.D., where t is the hour angle, *i.e.*, the distance of the body from the meridian. The fraction $\frac{2 \sin^2 \frac{1}{2} t}{\sin 1''}$ is calculated in Table V., for all values of t up to 20 minutes of time, and as this may be on either side of the meridian, so the observer may make his observations any time within 40 minutes of time, instead of being confined to one observation at a precise moment. This method is of great use when the observations have to be made with a sextant—in which case the observations need only be made upon the sun's lower limb—and the hour angle t will be obtained by applying the error of the clock to the observation, and deducting this from the mean time of apparent noon. In the following example an alt-azimuth instrument has been used, and the angle t is formed as shown in the foot-note.

Example.—Observations made at Malikpur on sun's lower and W. limbs, November 21st, 1866. Barometer, 29·3 inches; Thermometer, 85°.

Observed altitudes			Times.			Hour angles *			Value from Table V.
°	′	″	h.	m.	s.		m.	s.	s.
39	59	00	11	00	40	E.	5	03·7	50·3
39	59	20		01	53·5	E.	3	50·2	29·9
39	59	40		03	07	E.	2	36·7	13·4
39	59	40		04	11	E.	1	32·7	4·6
40	00	00		05	12	E.	0	31·7	0·6
39	59	40		06	06	W.	0	22·3	0·8
39	59	20		06	54	W.	1	10·3	2·7
39	59	20		07	57	W.	2	13·3	9·7
39	59	00		08	48	W.	3	04·3	18·6
38	58	40		09	48	W.	4	04·3	32·5
39	59	20	Mean of observed altitudes.					Sun	162·6
								Mean	16·26

− 01 02 5	Corrections for refractions and parallax.
+ 16 14·05	Semi-diameter.
40 14 31·55	True alt. of sun's centre.
+ 17·36	Correction to meridian.
40 14 48·31	Meridian altitude.
49 45 11·09	Zenith distance.
− 19 53 45·80	Decl. South.
29 51 25·29	Lat. of Malikpur.

h. m. s.	
11 45 58·66	Mean time of appt. noon
01 09 07	Mean time of semi-diameter passing meridian.
41 24·00	clock error, *slow*.
11 05 43·7	clock time of transit.
29 52 00·00	approx. latitude.

Cos approx. lat.	$= \bar{1}·9381126$
Cos decl.	$= \bar{1}·9732267$
Cosec. zenith dist.	$= 0·1172680$
Log 16·26	$= 1·2111205$
Log 17 36 secs.	$= 1·2397228$†

*These hour angles are obtained by applying the error of the clock, and also the mean time of the semi-diameter of the Sun passing the meridian, to the observed times:—the several differences between those corrected times and the mean time of apparent noon are the hour angles.

† When a star is observed, 0 0023715 must be added to this logarithm on account of the chronometer keeping mean solar time and not sidereal time.

Example.—Star γ Centauri observed on 17-5-1907 (compare the latitude observation on Polaris).

		h. m. s.	
R.A. of star	=	12 36 25	from N.A.
S.T. at L.M.N.	=	3 35 20	
Sid. Int. between transit and L.M.N.	=	9 01 05	
retardation		1 29	
Mean time interval between transit and L.M.N.	=	8 59 36	
Watch or clock error	+	38 58	
Watch time of transit	=	9 38 34	

Difference of time from 9 *h.* 38 *m.* 34 *s.* (m. s.)*	Table ($\sqrt{}$) values $\left(\frac{2 \sin^2 \frac{1}{2} t}{\sin 1''}\right)$
4 43	44
1 22	4
2 17	10
6 33	84
9 35	180
12 20	299
15 20	461
17 31	602
	8)1614
	210·5 = *m*.

Since there were 8 observations taken then *m* the mean = 210·5.

		° ′ ″	
Mean observed Z.D.	=	66 57 10	
Refraction		+ 2 04	
Approx. Z.D. (†)	=	66 59 14	on meridian.
N.P.D	=	138 27 11·4	(90 + δ since star is in the south or has a declination S).
Approximate colatitude	=	71 27 57·4	
∴ approx. latitude		18 32 02·6	
log cosec approx. Z.D.		0·0360150	
log sin N.P.D.		$\bar{1}$·8216655	
log cos approx. latitude		$\bar{1}$·9768701	
log *m*.		2 3232521	
		2·1578027	= 2′ 23 8″†

* These values are obtained from the record of observations on page ~~111~~ 97 and represent differences between the true watch time of transit and time of observation that is 9*h*. 38*m* 34*s*. − 9*h*. 33*m*. 51*s*. = 4*m*. 43*s*, etc.

† The approx. ZD is too large ∴ N.P.D. − approx. ZD = colat. is too small ∴ 90 − colat: is too large = (approx.) latitude too large hence the correction is always minus

$2' \, 23{\cdot}8''$ represents the quantity (less than the approx. latitude) which is required to reduce the approx. to the correct latitude.

$$\therefore \text{ latitude} = 18^\circ \, 32' \, 02{\cdot}6'' - 2' \, 23{\cdot}8''$$
$$= 18^\circ \, 2\overset{9}{2}' \, 38{\cdot}8''$$

and this value should be balanced against one on Polaris (2nd result); therefore the mean latitude is $18^\circ \, 29' \, 38{\cdot}72''$.

73. **Longitude.**—Longitude is the difference between the meridians of two places measured in arc on the equator.

As the apparent daily motion of the stars is uniform, and in circles parallel to the equator, the time elapsed between the transit of a star on the meridians of two places is evidently proportional to the arc between them, *i.e.*, to their difference of longitude. Hence, time may be taken as measure of longitude: and astronomically speaking, usually is so employed. As there is no fixed point on the equator from which to measure longitude, many nations choose the meridian of their own observatory as a zero point.

If then we know the local time of any place, and also the local time of the place we are at, the difference between these would evidently give the difference of longitude of the places in time.

1*st Method.*—The simplest way to determine this would be to send a chronometer, set to the local time of the first place, to the second, and then determine its error compared with the local time of the place by any of the previous methods. For small distances this is very accurate, and at sea is used throughout a whole voyage; but, as the rate of a chronometer, when travelling, differs from its rate when at rest, any error in the chronometer's rate will affect the deduced longitude. As one cannot, moreover, always send chronometers about from place to place, it is useful to know other methods.

2*nd Method.*—It is evident from the above that if a signal, such as the flashing of a light, of a rocket, etc., or some peculiar occurrence in the heavens which would present absolutely, the same conditions of observation at the same moment to all parts of the world, could be observed at several places, and the local time noted, the differences between the observed times would evidently be their difference in longitude. The eclipses of Jupiter's satellites are almost the only available celestial phenomena fulfilling these conditions; they are tabulated in the "Nautical Almanac," and furnish a very accurate method of determining the longitude; as however a telescope of much greater power than any that the engineer is likely to possess is required, so that the immersions

emersions, transits or shadows of the satellites may be distinctly noted, this method cannot always be employed in practice.

3rd Method.—The moon's motion in right ascension is so rapid (being 360° in about a month), that the interval between its transit over the meridians of two places, differs considerably from that of a star over the same places; this difference is the change of right ascension of the moon in that interval, and if that interval be not great (*i.e*, if the difference of longitude be small), it is proportional to it. As the change of the moon's right ascension is not uniform, this is not quite true when the longitude is great as in India, and the calculation on that account becomes more troublesome. The correction to be applied is, however, small, and being troublesome, may be neglected, unless great accuracy be required.

The right ascension of the bright limb of the moon is given for both upper and lower transits over the meridian of Greenwich for every day of the month; also its change in right ascension for one hour of longitude; * also the right ascension of four tolerably bright stars of nearly the same declination, which transit at about the same time at Greenwich; they, therefore, culminate at nearly the same altitude and time with the moon, and the corrections for refraction and instrumental errors are nearly the same for each; these stars are called *moon culminating* stars.

It is evident from the above, that if a transit of the moon's bright limb be observed, and also that of one of the above stars, and the difference of the times compared with the similar difference given for Greenwich, the difference of these differences will be the change of the moon's right ascension due to the longitude of the place, cleared of chronometer error; whence the longitude can be deduced as above.

Note.—If the chronometer error and rate be very accurately known, it is not necessary to observe the transit of the star also, but always advisable to do so.

In finding the time when the moon's transit may be expected to take place, the right ascension given in the Nautical Almanac must be corrected (when the longitude is so great as it is in India) to allow for the change of right ascension due to the longitude of the place, or else the transit will probably be ost. It is sufficient to know the longitude approximately for this purpose.

* This is the change in right ascension of the bright limb and is therefore cleared of the effect of change of semi-diameter.

Example.—To determine the longitude of Roorkee by observations on the transits of the moon's bright limb, and *a* Scorpii (Antares) on June 18th, 1864.

عقرب

Transit of *a* Scorpii. Observed times.				Transit of moon's bright limb. Observed times.		
H.	M.	S.		H.	M.	S.
10	17	21·5		10	36	23
		42				43·5
		62·5				64
		82·5				84·5
		102				104·
10	17	310·5	Sums.	10	36	319
10	18	02·1	Times of transit over mean wire.	10	37	03·8
				10	18	02·1
			∴ Difference at Roorkee in mean solar equivalents		19	01·7

	M.	S.	
Conversion into sidl. equivalents.	19	03·12	
		01·00	
		00 70	
	19	04·82	Sidl. interval between transits of moon's bright limb and star at Roorkee.

	H.	M.	S.	
R.A. of moon I.U. at transit at Greenwich on 18th June, 1864.	16	53	38·16	
R.A. of *a* Scorpii	16	21	08·46	
		32	29·70	Sidl. interval do, do, at Greenwich.
		19	04·82	Do., do, at Roorkee.
Change in R.A. of bright limb of moon, due to long of Roorkee		13	24·88	

		secs.
But var. of moon's R.A. in 1 hour of longitude at upper transit at Greenwich, 18th June, 1864	= +	155·71
Do., do., do., do., do., 12 hours previous	=	152·71
Change of variation in 12 hours	= +	3·00
∴ Change of variation in 5^h 10^m approximate longitude, East	= −	~~1·29~~
∴ Variation of R.A. of moon's bright limb in 1 hour at Roorkee	=	154·42
Do., do., do., Greenwich.	=	155·71
Do., do., in 1 hour half way between	= +	155·06

but total change = 13^m $24{\cdot}88^s$,

$$\therefore \text{Longitude of Roorkee} = \frac{13^m\ 24{\cdot}88^s}{155{\cdot}06} = \frac{804{\cdot}88}{155{\cdot}06}\text{ (hours)} = 5^h\ 11^m\ 26^s, \text{ East, nearly.}$$

4th Method.—The distance of the moon's bright limb from several bright stars and planets is given for every third hour, for every day in the month on which the moon is visible, in the "Nautical Almanac."

If a similar distance be observed at any other place, a comparison of the two will enable us to determine the longitude of the place, by a calculation similar in principle to the above.

The advantage of this method is that a sextant, an artificial horizon and a chronometer, alone are required.

It can, therefore, be used at sea, where it is, in general, the only available method, independent of the chronometer showing Greenwich mean time. It is not, however, a satisfactory mode of dealing with longitude, so no calculations by this method are shown here.

* * * * * *

A few interesting facts concerning the solar system.

Name	Diameter in miles.	Density, earth as 1.	Mass, Sun as 1.	Distance from sun in millions of miles.	Period of revolution in days.	Velocity in orbit, miles per hour.	Velocity of rotation at equator, in miles per hour.
Mercury ...	3,030	1·24	$\frac{1}{4,865,751}$	36	88	105,330	386
Venus ...	7,700	0·92	$\frac{1}{401,211}$	67	225	77,050	1,010
Earth ...	7,918	1·00	$\frac{1}{314,760}$	92·8	365¼	65,533	1,040
Mars ...	4,230	0·52	$\frac{1}{2,546,247}$	142	687	53,090	628
Minor Planets ...	...	...	...	250 to 500	...	...	...
Jupiter ...	86,500	0·22	$\frac{1}{1,046}$	483	4,332	28,744	27,985
Saturn ...	70,000	0·12	$\frac{1}{3,496}$	900	10,759	21,221	21,538
Uranus ...	31,500	0·18	$\frac{1}{24,899}$	1,800	30,687	14,963	10,921
Neptune ...	34,800	0·17	$\frac{1}{18,780}$	2,800	60,181	11,958	?
Sun ...	865,000	0·25	...	...	...	...	40,407
Moon ...	2,163	0·63	$\frac{1}{21,490,744}$	...	...	2,273	10

If the sun were a ball nine feet across, our earth would, in proportion be the size of a one-inch ball and at a distance of 323 yards from the sun. The moon would be a speck the size of a small pea, thirty inches from the earth. Nearer to the sun than the earth would be two similar

specks, the planets Mercury and Venus, at a distance of a hundred and twenty-five and two hundred and fifty yards respectively. Beyond the earth would come the planets Mars, Jupiter, Saturn, Uranus and Neptune at distances of 500, 1680, 3000, 6000, 9500 yards respectively. (The "Outline of History" by H. G. Wells).

One "*parsec*" or the parallax of one sec is equal to the distance away when the *diameter* of the earth's orbit subtends an angle of one second = 326 light years.

To have some conception of the magnitude of the cosmos and taking a "parsec" as an unit the diameter of the cluster of Hercules = 308 parsecs that of Magellan's cloud 340 parsecs or 110,000 light years.

As an illustration of the parsec it is accepted that a hair subtends one second at a distance of 20 metres or 61 feet.

New measurements just perfected by Dr. Francis Pease at the Carnegie Observatory at Mount Wilson California give the following for Antares (α Scorpii). The diameter of this star is given as 400,000,000 miles or four and a half times the mean distance which separates the earth from the sun. The star Mira in Cetus takes second place instead of Betelgeuse (α Orionis) the diameter of which is given as 250,000,000 miles or about 25% greater than Betelgeuse. The light from Mira takes 160 years to reach the earth and is 26 million times greater than that of our own sun. These comparisons however are small compared to the nebula in Andromedæ which is said to be distant at not less than 950,000 light years. In other words the light reaching us is that which left Andromeda nearly a million years ago.

For further useful information *see* "Whittaker's Almanac" any year.

74. **Sun-dials.**—As it falls to the lot of many engineers in India either to have to set up a sun-dial or to get one repaired, it may not be out of place here to see how they are constructed. They are two kinds in common use, the *horizontal* and the *vertical*; but dials can be adapted to a wall having any slope, in which case, however the calculations are not so simple.

A *sun-dial* is a surface, generally a plane, on which a system of lines is drawn in such a manner, that the coincidence of the shadow of a straight rod or edge with any of them, points out the hour of the day in apparent time. The straight rod or edge is called the *stile* or *gnomon* of the dial, and the system of lines, *hour lines*; and when the stile is the edge of a plate, the latter is called a *plate stile.* The plane of the plate stile is generally placed perpendicularly to the plane of the dial, and its intersection with the plane of the dial is called the *sub-stile.*

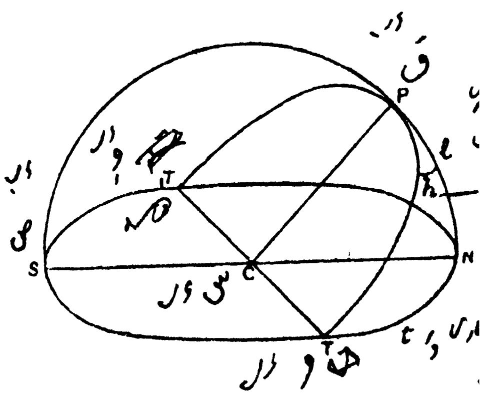
If a pole were set up in continuation of the earth's axis, its shadow would apparently move all round it in the course of a day's revolution of the earth, and mark the time on any graduated circle round its foot.* The stile of a sun-dial is always placed parallel to the earth's axis, and though owing to its distance from the pole, it describes a circle round the axis, its shadow practically does the same as if it were actually at the pole. If the dial plate were perpendicular to the stile, the hour lines required would just be at even distances of $\frac{360}{24}$ or 15° from each other all round, but in any case whatever be the inclination of the dial plate, hour lines are simply the intersections with the surface of the dial of planes passing through the stile, which, with the plane of the meridian, are inclined to one another at an angle of 15° in succession. The sun is always raised above his true place by refraction; and the effect of this would be sensible when he is low down, but it is lost in the general indistinctness of the shadow when he is high.

When the plane of a dial is horizontal, it is called a *horizontal dial*; when it is vertical it is called a *vertical* or *an erect dial*; and when the dial is both vertical and perpendicular to the meridian, it is called a *prime vertical dial*. Dials may be constructed by means of a terrestrial globe, by dialling scales, or by stereographic projection; but the most accurate way embraces the principles of spherical trigonometry.

75. *To construct a Horizontal Dial.* Let SNT be the plane of the dial extended to cut the celestial sphere, P the pole, SPN the plane of the meridian, and CPT the plane of an hour circle.

That is, that at that particular time the sun is in the plane CPT_1 (higher or lower on the arc T_1P according to the time of year), and therefore casting CT as the shadow of the stile, CP, and marking off the arc NT round this circumference of the dial from the noon position of the shadow at CN. CP is the stile, and CN the direction of the sub-stile. PN is the latitude. T_1CT is the hour line corresponding to the meridian, which gives the hours of the

Fig. 23.



* This time will be the time as given by the true Sun and not that of the mean sun so that Sun-dial time must always be corrected ± Equation of time and again for longitude west or east of the standard meridian to obtain standard time of the place (*see* para. 64).

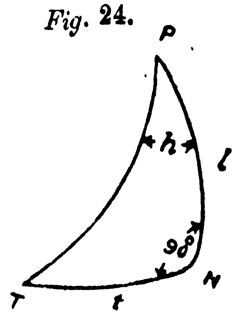
same name in the forenoon and afternoon, as for instance, five o'clock in the afternoon and morning ; also SN is the hour line of twelve.

Let l = PN, the latitude of the place,

h = angle TPN, the hour angle in degrees,

and t = NT, the distance in degrees of the hour lines from N ;
then in the triangle PNT, having the right angles at N –

Fig. 24.



$$\sin l = \cot h \times \tan t$$ (*see* Napier's rules of circular parts, para. 61).

$$\therefore \tan t = \sin l \tan h,$$

or $$\log \tan t = \log \sin l + \log \tan h - 10.$$

For any given latitude = l, the above equation will give the values of t, when $h = 15°, 30°, 45°$, etc.

Example.—Suppose a horizontal dial is required for Roorkee, latitude 29° 52′.

Angular distances of the hour lines from noon will be—

For 1 P.M. or 11 A.M.	$\tan t = \sin 29° 52' \times \tan 15°$,
7° 36′	$= \tan 7° 36'$,
For 2 P.M. or 10 A.M.	$\tan t = \sin 29° 52' \times \tan 30°$,
16° 2′	$= \tan 16° 2'$,
For 3 P.M. or 9 A.M.	$\tan t = \sin 29° 52' \times \tan 45°$,
26° 28′	$= \tan 26° 28'$,

and so on.

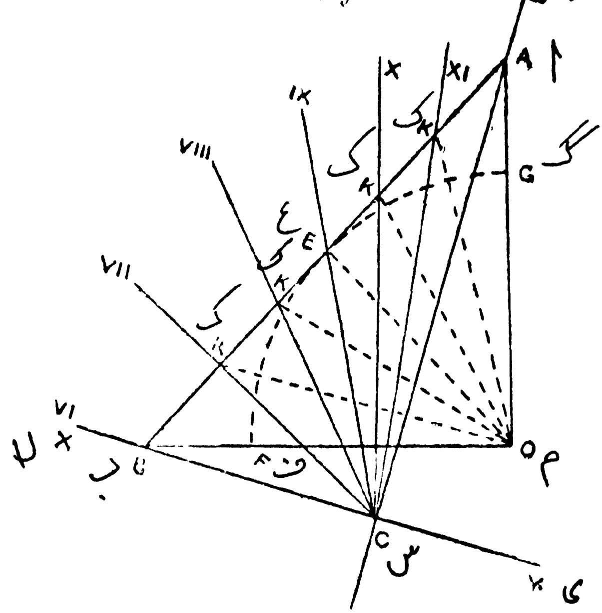
On a brass disc of convenient diameter, mark off a diameter to represent the north and south line, and on each side of it mark off the hour lines at the estimated angles, *i.e.*, 7° 36′, 16° 2′, 26° 28′, and so on. To this disc affix the gnomon, also of brass, the slant side of which must make an angle of 29° 52′ with the plane of the disc. Now, by means of a spirit level, carefully level in all directions some place, not liable to be shaken, so as to receive the metal disc. Then, having found the meridian by one of the methods described, carefully make the N. and S. line of the disc coincide with it, and then finally secure the disc to its stand.

It should be noted that in the above the gnomon is supposed to be a line, so that its shadow is cast by the centre of the sun, but if as is usually the case, the gnomon is a plate, then the shadow of its edge is cast by the highest point of the sun, and therefore the shadow is about one minute too *slow* before noon, and the same too *fast* after noon.

To read the sundial add or subtract this correction also add or subtract correction for equation of Time and finally correct difference of Local and Standard Mean time.

A sun-dial can be projected graphically by the following method

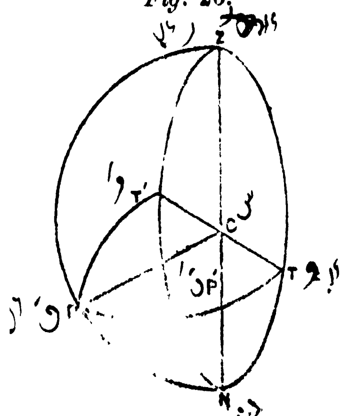
Fig. 25.



$\tan t = \sin l \tan h$ is the ordinary method of projection on XY and DC but if l = latitude and the angle ABC is projected as $\cot^{-1} \sin \lambda$ as for instance for latitude 29°, 52′; then natural sine 29° 52′ = ·4979833 = cot 63° 32′. Draw DC at right angles to XY and let the angle ABC = 63° 32′. Bisect AB in E and draw EO at right angles to AB and equal to EB or EA; with O as centre and OE as radius draw an arc FEG. Divide arc FEG into 6 parts, join O to each part and produce to meet AB in K, K′ etc. Join C to each K K etc., and these will be the hour lines required. Subdivide each sixth division of the arc FEG and proceed as above for smaller divisions of the hour.

76. *To construct a Prime Vertical Dial.*—Let TNZ be the plane of the dial produced to cut the celestial sphere, P the pole of an opposite name from the latitude, PZN the plane of the meridian, and CPT the plane of an hour circle, and CP the direction of the stile. Then PN is the co-latitude, and TCT′ the hour line corresponding to that meridian, which gives the hours of the same name T in the forenoon, and at T′ in the afternoon. Let l = the latitude, h = the hour angle in degrees, c = the co-latitude PN. Then we have in the spherical triangle PNT.

Fig. 26.



Cos $l = \tan t \times \cot h$, or log. tan t

$= \log \cos l + \log \tan h - 10$, where t is the angular distance of the hour lines in succession from the hour line of noon (*see* para. 61).

Example.—To construct a vertical dial for a place in latitude 31° 30.′ Angular distances of the hour lines from noon will be—

For 1 P.M. or 11 A.M.	$\tan t = \cos 31^\circ 30' + \tan 15^\circ$,
12° 52′	$= \tan 12^\circ 52'$ nearly,
For 2 P.M. or 10 A.M.	$\tan t = \cos 31^\circ 30' + \tan 30^\circ$,
26° 12′	$= \tan 26^\circ 12'$,
For 3 P.M. or 9 A.M.	$\tan t = \cos 31^\circ 30' + \tan 45^\circ$,
40° 27′	$= \tan 40^\circ 27'$,

and so on.

The angle formed by the gnomon with the vertical face of the dial will of course be 58° 30′ (= *co-latitude*).

In a horizontal and vertical dial, the *elevation* of the stile is respectively equal to the *latitude* and *co-latitude* of the place; for in figure 24 showing a horizontal dial, PCN is the latitude; and in figure 25 showing a vertical dial, PCN is the co-latitude.

The inclination of the plane of an oblique dial to the horizon or plane of a horizontal dial is called its *inclination*; and its inclination to the prime vertical is called its *declination*.

77. Hence, if a *horizontal* dial, constructed for a given place, is carried to any other place on the *same meridian*, and placed in a plane parallel to the horizon of the former place, that is, parallel to its first position—it will be an *inclining* dial for the latter place; and its inclination at this place will be equal to the difference of latitudes of the two places. Also the elevation of the stile of an inclining dial at any place is equal to the sum or difference of the latitude and inclination. Hence, to construct an inclining dial at a given place, find the latitude of the plane to whose horizon the plane of the inclined dial is parallel and construct it as a horizontal dial for this place, and it will be the required dial; the latitude of this latter place will be the sum or difference of the latitude of the given place and the inclination.

In this manner a dial constructed for one place may be used at another place. For instance, suppose a horizontal dial made for Delhi (lat. 28° 36′) is to be set up in Lahore (lat 31° 34′) so as to show Lahore time. The difference in latitude is 2° 55.′ The stand, to receive the dial, will not be horizontal, but will be inclined upwards towards the north at an angle of 2° 55′ with horizon. It will then show Lahore time.

At the equator, the stile and sub-stile of a horizontal dial coincide; the stile then has to be placed above the plane of the dial, and parallel to the meridian. The hour lines are now all parallel with north and south line, and distant from it by the tangent of the corresponding hour circle's inclination to the meridian, the height of the stile being the radius.

Let S = the height of the stile above the dial of the stile; h, the hour angle in degrees; and t, the perpendicular distance of the corresponding hour line from that of noon; then—$t = S \tan h$.

The stile of a prime vertical dial would evidently be perpendicular to its plane, and the hour lines would make angles of 15° with each other in succession.

CHAPTER IV.

ENGINEERING SURVEYS.

78. The surveying operations described in the foregoing chapters may be considered to have as their general object the completion of a *map* of the country surveyed, more or less accurate according to the time and labour bestowed and the means employed.

Engineering Project.—When however, a survey is directed to the special object of preparing an *Engineering Project*, the preparation of the map must be regarded as subsidiary to the main design,—and the nature of the process employed must be regulated primarily towards the collection of data, which will be necessary in drawing out the project in proper detail.

The ordinary projects which are likely to occupy the attention of engineers in India are those for roads, railways and canals. Projects for drainage and water supply may also be occasionally required and even harbour works and light-houses; but these last refer to a special branch of surveying, called Marine Surveying, which will not be discussed in this Manual.

In the projects mentioned above, the necessary surveying operation must depend a good deal on what maps already exist of the country affected. Many parts of India have now been so accurately surveyed and mapped, that it may be quite sufficient to trace off the acquired area of country, to an enlarged scale, if necessary. If the enlarged map does not give the necessary details, such details may be readily interpolated by the theodolite, or prismatic compass, or even in the act of running the necessary lines of levels if stadia is used in conjuction with a planetable. But as very few parts of the country have yet been covered, even meagrely, with a net-work of levels, it will in almost every case, be necessary to run such as are required.

If, however, no map exists, or none from which leading points may be transferred to an enlarged map with sufficient accuracy, it will be necessary to direct the preliminary surveys to the construction of such a map.

79. **Preliminary surveys**.—The surveyor who is called upon to prepare a map of a given tract of country within a certain time, may often be at a loss to know the best method of starting the survey; whether to fix a series of points trigonometrically by a net-work of triangles, or else to

work by means of a number of closed traverses, and plot according to Gale's method. If the country is at all hilly, the first method is perhaps the best, and in the end the most expeditious, as then the errors are not cumulative, but are adjusted between station and station. But if the country is level, running traverses without first fixing any points will be found the most expeditious, and in the plains of India sufficiently accurate for all practical purposes. If a trigonometrical survey were resorted to, greater dependence could be placed upon the accuracy of the result, but that amount of accuracy is only obtained by much greater expense of time and labour, owing to the necessity of building high stations from which to observe. In traversing, the errors due to chaining are certainly cumulative throughout the entire circuit, but when the country is moderately level, these errors can with care be reduced to a minimum, and are easily allowed for in the calculations. The details of the survey in either case must be filled in as already explained in Chapter VI. Part I.

The least area of country which must be thus surveyed will be determined by considering what is the least area that will be affected by the project under consideration. Thus in the case of an irrigation canal running on the watershed of the country, the boundaries of the survey will clearly be defined by the two main water-courses on the right and left of the watershed line by the highest point on the river from which the canal is to be fed and the lowest point into which it is proposed to tail it. In the case of a railway or road between any two places A and B, the boundaries will be determined by the greatest possible lines of divergence from the straight line AB, *i.e.*, by the most circuitous route to the right and left of that line which it is possible the road might, with any advantage, be made ; and so on for other cases.

As to the general details which will be required—in the case of a railway or road the following may be enumerated :—The position and comparative size of all towns or villages affected thereby—(if the number of the population can be ascertained it may be written down in the map) —the exact course of any stream which will require bridging between the two extreme points where the bridge would probably have to be made—the cultivated, culturable, or forest land that would be traversed by the line—the position of brick-fields, stone quarries, forests or other materials that might be used in the construction of the line—the position and size of any swamps that might require to be crossed or perhaps drained.

Levels.—The map thus far having been completed, the lines of levels that will be required must next be run, and plotted on the map, the reduced levels being written in at every bench-mark and point of any importance,

or where there are none such, every 5th station or so, so as not to overcrowd the plan with figures.

Protractor,—A half sheet of "writing imperial" paper, 15″×22″, having a circular protractor of 6 inches radius lithographed in the centre of it, is found to be a very convenient size to protract preliminary lines of levels. The circular protractor is divided by large division into degrees, still larger marks show every 5th degree, and by smaller into ¼ degrees, but these marks are not numbered. The surveyor, knowing exactly from his previous field-work how his lines of levels run, draws his own north and south lines through that diameter of the protractor which he considers will make his work take somewhat of a diagonal direction across the paper, and then numbers the scale right round for his own convenience. In this way one of these sheets will hold a week's work of some 24 or 25 miles of line levelling, and all the necessary side detail, if drawn to a scale of 1 mile to 1 inch. A specimen sheet with the protractor attached, and the protraction of some levels is here given, *Plate* (XI) but some of the detail has had to be omitted owing to the reduction of scale.

A plan and section of a circuit of levels is here given (*Plate* XII) to show the amount of information which should be given, and the student should make a careful inspection of it, for few surveyors' plans and sections contain the whole of the information they should contain.

80. **Road**.—In the case of a road in open level country, it will be enough to level down the line decided on, or along the trial line previous to the actual one being fixed—the streams to be crossed must also be levelled along and the necessary cross sections taken so as to enable the proper calculations to be made with regard to the waterway, and the reduced levels of flood lines must everywhere be carefully ascertained, to enable the proper height of embankment to be determined. Cross levels will also be required, at points where the line turns and where a curve will be necessary,—also at points where a temporary divergence from the straight may be advisable in order to save work—as for instance, in crossing through a morass or over a hill.

Hill Roads.—To select the trace for a road in a hill region requires much more care and attention. Before anything is definitely decided upon, the several possible lines between the extreme points must be considered, and particular attention should be paid to the mean obligatory points on each line. A cardinal principle to be remembered, also, is that the ascent or descent should be as uniform as possible, the gradients in the opposite direction being reduced to a minimum.* Again, an obstacle—such as a steep cliff, chasm, etc., which at first sight may appear impassable except

* This can be modified to a certain extent as in modern practice for cart traction it has been thought a few pieces of level eases the strain and gives a rest to the animals.

at inordinate cost, may in reality prove quite otherwise, for in order to avoid it the numerous minor difficulties encountered may in the end prove more expensive. For obvious reasons also deep cuttings into the hill-side should be avoided as much as possible, but when the slope of the hill-side is considerable, the trace of the road should be so selected that the whole of the width of proposed road should be cut out of the solid hill-side.

In commencing a survey for a hill road,* the rough obligatory points are usually assumed on the several ridges to be passed, and as the length of the trace in the intermediate valleys is most deceptive and generally much greater than estimated, the relative position of our assumed obligatory point from the next should always be calculated by using a gradient steeper than that required to be worked to. For instance, supposing a road is to be laid out on a gradient of 1 in 20, a position on one ridge is known, and it is required to estimate where the trace may cut the next ridge. The intermediate distance is so difficult to approximate to, and so generally under-estimated, that the point on the next ridge is easier found by applying a steeper gradient—say 1 in 17 or 1 in 18—to the estimated length of intermediate road than by adding approximately to the estimate. Again, it must be remembered that when the trace of the road is settled, *the resulting gradient on the completed road will work out steeper than that used in laying out the trace.* The original trace must, therefore, be somewhat easier than that finally required; and though it is difficult to say what allowance should be made, as it varies with nature of the ground, it may be assumed in a general way that a

slope of	1 in 32	works down to a slope of	1 in 30
,,	1 ,, 22	,, ,,	1 ,, 20
,,	1 ,, 16	,, ,,	1 ,, 15

Zig-zags in mountain roads should be avoided as much as possible, as they frequently involve incessant repairs; if used they should not be run through rotten ground, or across drainage, but they are less objectionable if they can be arranged so that the drainage of each length can be thrown off at the turning points clear of the road below. The Ghat tracer, Abney level and De Lisle reflecting level are all used in tracing out hill roads and are sufficiently accurate for cart and coolie traffic routes.

81. **Railway.**—A survey for a railway is very similar to that for a road, but it is more elaborate, in that greater attention must be given to the gradients, and the several lengths of straight must be connected together by regular curves.

A revised and complete set of "Rules to be observed for the preparation of Railway Project" to be submitted for the sanction of the Government

* The author assumes here those so called hill roads before they were converted in some places, to pass motor traffic of all descriptions.

of India was issued in 1893, corrected up to 1918 and contains very detailed instructions.

One or more trial lines are generally run before the actual direction of a line of railway is decided upon, and the amount of accuracy bestowed upon the surveys of such trial lines is only sufficient to obtain a fair approximation to accuracy, and to give a reliable comparison between the rival lines. In the preliminary surveys therefore it is not usual to run in regular curves except in difficult country, nor to take more levels than are sufficient to admit of an approximate estimate of the earth work being made. Therefore to make a survey for a trial line of railway, all that is necessary is to run a traverse with a theodolite from one obligatory point to the next and then run a line of levels over the same line.

Owing to facilities for vision, the engineer in running the trial line is apt to put the turning points on the several ridges crossed, but this should be studiously avoided if possible, as if the line is selected for the permanent line, it will have to be re-run, or else the cuttings through the hills will all be on the curve.

It is usual for the surveyor who runs the traverse to put in pegs or marks every 300 or 500 feet along the line of traverse in open country and every 100 feet in broken or hilly ground, in order that the leveller who follows may not require to do any chaining. These pegs are usually numbered consecutively, the surveyor continuing the length of the chainage throughout from the beginning, rather than commencing a fresh chainage at each new departure from the previous straight line. It is obvious in this trial survey that the levels are taken along the tangents rather than along the curves, and also that the line of railway is represented as slightly longer than it would be if really located, but the results are quite accurate enough for the purposes of the trial survey. Also in a trial survey the amount of detail to be shown may be reduced to a minimum. Of course more detail of the line is required in difficult country and through towns and villages, but no great accuracy is aimed at as the results desired are in the first instance only comparative. In a trial survey, therefore, all the detail required is to show where obstacles occur near the traversed line, and also the course of rivers and streams for a few hundred yards on each side of the line.

When, however, the course of the railway is decided upon, the work required from the surveyor who has to locate the line is much more delicate. Sufficient detail of the country on either side of the selected line has to be shown, in order that possible minor deviations may be made if it is found that the estimates work out unsatisfactorily and demand a further

investigation. In locating a line it is therefore usual to lay out the several lengths of straight accurately, and to determine the most convenient curves by which to connect them; the line is then traversed over in the ordinary way, and the curves put in as the work proceeds. The chainage is usually continued throughout from the commencement, and pegs put in the centre line at every 100 feet, for the convenience of the leveller who has to follow the man with the theodolite. The detail of the neighbouring country may be also put in by the man who runs the traverse, but it is better to leave all that to an assistant who follows with a plane-table, on which the course of the line with the positions of the 100 feet pegs is accurately marked.

The smallest party required to locate a line of railway with convenient despatch is one Executive Engineer and two assistants. The responsible officer ranges out the line, conducts the traverse, and puts in the curves; one assistant follows with the level, makes an accurate section of the line and also takes all necessary cross sections; and the other assistant, with the aid of a plane-table and other instruments*, when required, makes a survey of the required belt of country, surveys a sufficient length of the course of all streams crossed, and does the needful check levels. With a large staff the above work can be sub-divided and greater progress made, and assistance is also at hand to run alternative lines through difficult parts, and to make more detailed investigations as river crossings, etc. If it can be arranged, it is advisable that one party be told off to locate a line of railway up to, say in India, 150 or 200 miles in length; and the work should be so regulated that the several officers engaged, work comparatively close to each other. Check levels should be constantly run, and no great advancement should be made by the traverser ahead of the levelling operations until the levels are accurately checked. If a man has of necessity to check his own levels, he should do it in the *opposite* direction, and only touch on the original line at bench-marks and other obligatory points, but it should be so arranged that the check levels are run independently of the leveller.

As the location of a line of railway usually follows quickly on the trial surveys, it is only necessary to make such preliminary marks as will enable any of the trial lines to be easily found. This can be done by blazing trees, making marks on buildings, etc., by which the line passes close, and by making a semi-permanent mark where the several tangents

* The cross section work and also the detail is easily put in by an assistant working the level taking stadia readings and shooting in staff positions from his plane-table placed as close as possible to his level.

meet; and if careful sketches and notes of these marks are also entered in the field-books, no difficulty should be experienced in picking up a line again. With a located line the marks must be much more permanent, and it is usual to show by a permanent mark in the actual line each successive mile-post and the ends of every curve. It is also of great assistance if the intersection point of the tangents is also permanently marked. The centre line is also shown by a continuous cut along the surface of the ground about 6 or 9 inches in depth, a mark generally sufficiently accurate in the plains of India to lay off the dimensions of the ground required for the railway.

82. **Canal.**—The kind of survey required for a canal will depend very much on the nature of the country and the magnitude of the work required to be constructed. In an undulating or hilly country the approximate direction of the irrigation channel is apparent at once, and the positions of the minor distributing channels in all cases vary with local requirements; but a project for a large irrigation canal in the plains of India requires a very extended survey.

For such a project, besides the details required for an accurate representation of the country to be traversed, it is necessary to cover the map with a network of levels; and this is generally done by running a series of roughly parallel lines of levels at distances of about a mile apart, and as nearly as can be estimated at right angles to the general watershed of the country traversed. If these lines of levels are connected at their extremities by other lines of levels the work will be continuous, and each part serve as a check to the reminder. The best line for the canal will probably at once be evident from the contours deduced from this network of levels, and a fairly approximate estimate may be made without further fieldwork. But if any sand hills, ridges, or large shallow depressions occur, and which have not been sufficiently marked by the general lines of levels run, it will be necessary to run such further lines of levels, as will carefully represent the ground at these parts, if needed.

The line of the canal selected will generally run along the highest ridge or backbone of the country, and its alignment should be made as straight as possible. With any appreciable current the outer side of a curved channel will require to be revetted, and so add considerably to the initial expense, unless the curve can be made extremely easy.

83. **Canal Surveys.**—The following instructions, based generally on those issued by Colonel Crofton, R.E., when Chief Engineer of Irrigation in the Punjab, for the conduct of canal surveys, will give what further

information is required. They are applicable to all kinds of engineering surveys.

Trial levelling and surveying.—In addition to the levels of the country surface, a rough survey or reconnaissance is required, which should give information on the following points, *viz* :—Approximate sites of villages or towns, lines of drainage, roads, railways, old water-courses canals, channels (main or rajbahas), edges of high or "bangar" land remarkable buildings, wells, nature of soils, crops, trees, etc., position of stone or kankar quarries, etc. The places between which roads run and their bearings (if regularly lined out), should be noted ; if on embankment the level of the top surface should be taken. The bearings of regularly lined-out canal channels or irrigation cuts, and the level of their beds at points of crossing with cross sections at right angles to the direction of each, showing level of full supply are required.

84. *Water level.*—The level of the lowest point in the beds of streams where crossed ; with sections at right angles to their courses showing level of highest known flood, and date of its occurrence, if ascertainable ; the level of surface of water in rivers (noting date of observation) ; depth of water on lowest point of bed (if obtainable), and level of ordinary and highest known flood ; levels of floors of tanks and lowest points of large swamps should be observed and connected with the line of levels. The site of such sections taken off the line should invariably be connected with the traverse.

The waterway of all bridges or culverts met with on or near the line of levels should be measured ; and the levels of their floors or plinths of abutments, or the bed under the arches if there be no flooring, with highest flood mark, carefully noted.

Wherever a well is met with or used as a bench-mark, the level of the surface of the water should be noted : the depth below the bench-mark can be measured with sufficient accuracy by the chain. If water is being drawn from the well, the surface will in general be abnormally low, in which case the height at which it usually stands when not in use should if possible be ascertained. The quality of the water, whether sweet or brackish, should also be noted. These observations of the surface level of the springs should never be omitted, when opportunity offers ; it is a point of considerable importance.

The colour and description of the soils, whether sandy, clayey, etc., the presence of the white or brown efflorescence, known as "reh" or "kuller" should be noted.

85. **Drainage lines.**—A complete delineation of the drainage lines of the country being one of the primary objects of the survey, too great care cannot be taken in ascertaining their positions. They may be divided (excluding the large rivers) into two classes; the first easily recognizable by their size; well defined channels running in valleys at some depth below the general level of the country adjoining. Into these and the rivers, innumerable channels of the second class discharge themselves; the exact positions of which are not always to be detected by the level alone. They usually rise in jhils (swamps) lying close to the watershed, and their courses are marked by a series of jhils connected by intermediate low lands; a black, clayey soil, "reh," rank grass, and crops requiring frequent irrigation, such as sugarcane, cotton, etc., generally mark the places where water has lain or over which it flows in considerable quantity. No land of this description should be passed over without enquiry as to whether it is flooded during rain, and from what direction the water comes, and whither it runs off. "Reh" if contained in the soil, always rises to the surface where water has lain for any time and appears in greatest quantity during the cold season.

Large towns or villages will almost invariably be found situated close to lines of drainage, or to low ground where water collects after rain. In those parts of the country which are subject to extensive inundations, villages, especially small ones, will always be found to be situated on land out of the reach of the ordinary inundations; no reliance, however, can be placed on their position as regards extraordinary floods.

Sand hills, or very sandy soil, generally mark a watershed on the "bangar" or high land.

Where a nala or drainage lines is crossed, and the level of the lowest point of the bed is observed, great care should be taken to ascertain whether this point is on the general level of the bed; if otherwise, the difference above or below should be measured and noted.

Where drainages are met with, enquiries should be made as to their courses both above and below the line of levels, names of villages near which they pass, etc.; by thus observing them in each successive line of cross section, a very complete plan of the drainage of the country is obtainable, as well as connected series of levels along the beds of the outfall.

Similar information to that detailed above should be obtained with all levelling or surveying for distributaries, drainage projects, or any other work connected with irrigation.

In levelling for the longitudinal section of a river, the line should follow generally the main water channel, the stations being invariably on the bank or dry ground near the edge of the stream. The level of the surface of the water at intervals (noting date) of ordinary floods and highest known floods ; the position of top and foot of rapids (if any), and level of surface of water at each point to be noted. The depth of water to be measured in the deepest part of the channel where the surface level has been observed. Cross-sections at right angles to the direction of the river should be taken at intervals and connected with the series of levels showing the bed, surface of water, level of ordinary and highest known flood. The survey should show all minor channels and affluents (if any), and as nearly as possible the extent of land under water in high floods. The nature of the bed, whether boulders, sand, clay, etc., should be carefully noted.

86. **Bench marks** should be established at intervals of about 3 miles in *general*, and one close to the crossing of every large stream or line of drainage, but at a place not likely to be washed away ; also at the ends of each cross section or line of levels. Existing buildings to be preferred for the purpose.

All canal, road, railway, Great Trigonometrical Survey, or other bench-marks met with *en route*, should be connected with the line of levels.

87. **Admissible error in levelling.**—The error or difference in any circuit of levels ought not to exceed one foot per hundred miles linear.* *Small* errors arising from incorrect reading of the staff, not holding it vertical, high wind and such like, are inseparable from all levelling operations, but these will not be found to *accumulate* if the work be carefully done. A tendency, however, has long been observed, though as yet unaccounted for, to a small cumulative error in the direction of the levels ; but this is not found to affect practical operations materially. Where great accuracy is required such as in the proof levels of a canal channel, it is advisable to level twice over the same stations with the same instrument, the second series of levels being carried in the reverse direction to the first ; the mean reduced level of each station will be as nearly accurate as it is possible to obtain it.

A prismatic compass attached to the level will be found very useful in filling in details off the line of the series of levels. If the variation of the needle is not identical with that of the map employed, the bearings should be reduced to the meridian of the latter.† Most of the side measurements,

* Error in feet = Constant $\sqrt{\text{distance in miles}}$ where C = 0·1 (*vide* para. 102 Part I).

† Modern levels have slotted screw holes to enable the compass being cleared of Magnetic deviation.

where great accuracy is not required, may be made by pacing. Two and-a-half or three feet paces will be found most convenient as admitting of easy reduction to feet. Stadia measurements are now more generally employed and are of course accurate.

88. **Scale.**—The scale generally for protraction of levels should be 1 mile to 1 inch. For the section, the horizontal scale same as for the protraction; the vertical, about 100 times the horizontal. A larger or smaller scale may be necessary for special purposes; they should, however, be *always* measures or aliquot parts of the one-mile-to-the-inch scale and for working plans 400′, 200′, 100′ or 50′ to the inch.

On every protraction of levels, besides the heading, the following must never be omitted:—Date of the survey, name of the surveyor, scale and meridian line; the numbers attached to the several stations on the section to be identical with those on the protraction.

All details noted in the field-book should be transferred to the protraction or sections; a sketch and a short description of each bench-mark to be entered on the back or margin of the sheet in which its position is shown. The information is thus more accessible than if old field-books have to be searched for it.

If a map is to be compiled from levels or surveys taken with more than one instrument, it will be found best to protract the work done with each instrument, on separate sheets, to be subsequently transferred on to the map or have the compasses corrected as suggested in footnote.

89. **Running the traverse.**—After the position of the line, which may generally be assumed as the watershed, has been approximately determined by means of the cross sections, or otherwise an accurate traverse with the theodolite should be taken over it, including a survey of the ground for about half a mile in general, or further, if deemed necessary, on each side, which should give information on the following points, *viz*:—Features of the country, if irregular; streams, lines of drainage and swamps wherever met with; sand hills or ridges, towns and villages; wells; buildings, whether of masonry or mud; roads, whether regularly lined out or merely cart tracks—if the former, the bearings should be taken; places between which they run (whether tracks or made roads), and whether they are lines of traffic or merely village communications, should be carefully ascertained (this is useful afterwards in determining the sites of bridges); village boundaries, etc.; such minutiæ as the boundaries of fields are unnecessary; those of gardens may be useful; in fact, everything which is likely to be of assistance in determining the precise line, or that which it would be advisable to avoid if possible. A

survey of this nature, carefully taken, will generally admit of choosing a line which will not injure property or disturb existing rights in the least possible degree.

The accuracy of the traverse is the point to be chiefly looked to ; the distance between the stations on it should be as long as possible, less than a mile, as the probabilities of accuracy in observation are greater in the case of long than short sights, and the plotting is easier as well as more likely to be accurate. The sights to the station poles should be taken as in ordinary traverse surveying by the inward angle method which is an angular check in itself. To check the distances between stations fix on a well-defined point some distance to one side, say, a mile, and observe to it from every station from which it is visible. If the distances have been measured and plotted correctly, according to angles observed the lines will all meet in one point on the map.

The above paragraphs regarding traversing should not be adhered to by the surveyor who can fix a true meridian and whose work may be required many years hence. No reliance can be placed on magnetic variations which differ year by year and which are different also in every instrument. The best way to locate a line for present or future use is by a true meridian with inward angles and checks by observations for meridian at every 5 miles or so with the correction for convergency applied (*see* paras. 131 and 132 Part I).

90. **Stations.**—The stations may be marked on the ground by large pegs, about 3 feet long, driven well in. If their future identification is an object, and there is a chance of the pegs being destroyed or removed, a *ghurrah* or earthen vessel filled with charcoal, buried at some depth below the surface of the ground, will give the means of finding their sites again with sufficient accuracy for all practical purposes. The surest way, however, is to note their distance and bearings from any easily recognized and permanent objects, which are not likely to be disturbed, if such should be found sufficiently near for the purpose. It will be found most convenient to fix all stations on mounds or rising ground.

It will be found convenient also to have two descriptions of poles (*jhandis*) for setting up at stations to which observations are to be taken one for use in windy weather, mounted with a flag ; the other when the air is calm, with a small "moon" (made by covering a wooden hoop with calico), about 1½ feet diameter; as a flag when not flying free is scarcely more distinguishable at a distance than the bare pole. On the Revenue Survey, poles painted in foot lengths, white and black alternately, are employed, which makes them visible at a far greater distance than the common uncoloured bamboos.

The angles on this side surveys may be taken with a good compass, prismatic, or of any other description available; the actual bearings, as shewn by the instrument employed, being entered in the field-book, *i.e.*, no correction being made *in the field* for the variation of compass (if any). Villages should be traversed round, so as to determine their outer limits, but no interior survey is required. These should be connected with points on the main line of traverse; the correctness of the junction line may be tested by observing from several points on it to some objects in or near the village (such as a large tree, house, etc.,) which has been well connected with the boundary survey of the village.

As the choice of a good line and the actual lining out on the ground very much depends on the accuracy of the map, this should be placed beyond a doubt, if possible, before the line is chosen and marked on it. The time occupied in taking check observations and measurements in the field will be well repaid by the facilities afforded to the subsequent work by a really accurate plan of the country.

The position of the actual watershed near the line of traverse should be carefully ascertained and noted on the map.

To the above may be added a list of the maps and drawings generally required in an engineering project.

91. **Road.**—(1). *General map of country.*—The plan sufficiently broad to show the greatest amount of likely deviation. The scale of the plan will of course vary with the length of the road, but, as a rule, should not be less than 1 mile to 1 inch. (Standard map Survey of India).

The road should be laid down on this plan, which should show the lines and cross lines actually levelled, and as many reduced levels as convenient without crowding the plan. The positions of the bench-marks must be shown and numbered, and a sketch of the bench-marks showing, where the staff rested should be added in the margin.

(2) *A longitudinal section* along proposed road.—This should show the natural surface of the ground, and that of the proposed road, and also in columns just below these surface lines, the depth of cutting or height of embankment. If the ground is level, the horizontal scale may be similar to that of the general plan and the reduced levels entered at every 1,000 feet. The vertical scale should be at least 10 times the horizontal scale (*see* para 88).

If the ground is at all undulating, the horizontal scale should be adopted so as to show reduced levels at every 100 or 200 feet. The sections should also show the villages it passes through, the kind of cultivation, and the bearings of the different parts of the road written over

their respective portions, so that, in case the plan is mislaid, the section may in a manner supply its place. The height of water in wells, and highest flood lines of all water-courses, should be carefully entered. The stations should be numbered so as to correspond with those on the plan, and the horizontal distances should be marked.

(3) *The principal cross sections*—The same remarks apply.

(4) *Half section of road when in embankment*, showing the positions and widths of the metalled and unmetalled parts, the side slopes, drains, fencing (if necessary), etc.

Half section of road when in cutting, and complete section when partly in cutting and partly in embankment; both of these showing similar details.

(5) *Bridge site plan.*—If a river has to be bridged, a plan to a large scale of the course of the river for a considerable distance on both sides of the bridge site must be made so as to show *why* that site has been chosen in preference to any other.

(6) That part of the longitudinal section showing the passage of a river and the low land on either side, must be again drawn to a much larger scale, so as to show all the reduced levels. Any steep ascent or descent should also be shown on a large scale.

(7) Plans and sections of all *bridges* and *culverts* necessary; and these again must be accompanied by drawings of detail to a much larger scale.

(8) Plan, section and elevation of an *inspection bungalow* and *store-house* for tools, etc.

92. **Canal.**—The plans for a canal project will be similar to those for road with a few additions.

The survey should be of scale not less than 400 feet to an inch.

In the longitudinal section, the line showing the surface of the water will be shown, as well as the line of the bed of the canal.

Besides the above-mentioned drawings, there will be required plans and sections of the lock channels, the lock gates, main and minor distributing channels, dams, falls, inlets for surface drainage, escapes for the passage of flood waters, aqueducts, bridges, etc.

It would be impossible to give the full details of the operations required for the preparation of projects for canals and railways in a general Manual on Surveying, without unduly increasing the scope and expense of the work. Directions for the guidance of surveyors engaged on such highly technical work, will be found in the publications specially devoted to these subjects.

93. **Railway***.—The drawings for a railway project are similar to those for a road. Besides, however, there will be required detail drawings, to a large scale, of the permanent way and of the rolling stock. Plans and sections, etc. of the stations, sheds, engine house and water tanks will also have to be provided.

SCALES.—The following scales will be found very convenient for all maps and plans, and are generally used in the Public Works Department:—

For general maps—

Two or four miles to the inch.

For maps accompanied by sections—

According to the amount of detail required.

Map one inch to a mile—Index plan and section.

100 feet to the inch vertical.

Detail plans and sections—

400 feet to the inch horizontal.

40 " " " vertical.

For all plans of buildings, the scale will be either $\frac{1}{50}$, $\frac{1}{100}$, $\frac{1}{200}$.

94. **Useful Hints**.—The following hints may be found useful:—

1. When series of levels are taken over a tract of country, the plan and section of such levels should correspond exactly. If the scale is not too small, the measured distances between stations should be shown in *both*; the numbers of the stations, as shown in the field-book being given at every 5th station on the plan and section, with the reduced levels written on the plan in red ink. The situations of bench-marks should be shown accurately on the plan, and the reduced levels written clearly showing to what exact spot the numbers refer. Wherever the scale admits of it, the information given on the plan should be so full and complete, that the sections can at any time be drawn out from it alone; and, if the azimuths of the different lines be written on the section, the plan may conversely be laid down from the sections alone. These can be had from the Traverse form.

2. Where a line of levels crosses a water-course, the reduced level of the bed of such water-course should be shown—that of the water-surface (the date of observation being given)—of highest and ordinary flood mark, if discernible—and that of the top of the bank; and all these reduced levels should be shown on the plan.

3. In levelling, the staff, unless when placed on a bench-mark, or a pucka road, must always be held on a wooden peg, about 3 or 4 inches long, driven in flush with the surface of the ground; without this no confidence can be placed on the accuracy of the work. Local depressions and elevations should be avoided for peg positions.

*See rules for the Preparation of Railway Projects issued by order of the Government of India.

4. The level, if not absolutely impossible, should invariably be set up equidistant from either staff; errors of adjustment being thus completely obviated. If otherwise, it is to be borne in mind that when the line of collimation and the large bubble are in adjustment with each other, although not so with reference to the axis of the instrument, a correct result is obtainable by bringing the bubble horizontal at each sight or to contact position as in the India Pattern Level.

5. The ordinary distances from the level to either staff to be in even hundreds or half hundreds of feet, not aliquot parts of miles.

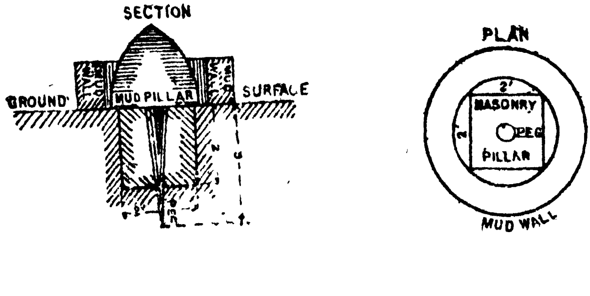
6. All observations for level *without exception* must be connected with some point of which the reduced level (from the common datum) has been previously, or will be, ascertained. Sections of rivers, nalas, etc., to be thus connected.

7. In *line* levelling, it will be found the simplest mode and least liable to errors to record collateral observations separately from the line series in the field-book. Such observations should be taken and recorded with reference to stations on the line.

8. Measuring chains (100 feet) should be kept of the exact length even for ordinary surveying or levelling. Their length when new should be verified daily at commencement and close of work against a steel tape or a standard chain kept solely for this purpose every new chain stretching considerably at first With a chain of good material some time in use this elongation is scarcely perceptible.

10. **Bench-marks.**—Invariably on masonry buildings, or other *permanent* structures. In the choice of their positions, security from injury and facility of identification are the points to be chiefly looked to. The sills of niches in, or plinths of masonry buildings are very suitable. In a well, the small niche (or *namah*) usually left for a slab engraved with the owner's or builder's name, if the sill be flat and even, is a very safe place. Where it is necessary to build a pillar for the purpose, some retired nook or waste land should be chosen as the site: the pillar, of masonry in lime cement, may be of this form—

Fig. 27.



A wooden peg about 3 feet in length being inserted in the centre of the masonry the top flush with its upper surface, on which the levelling staff is to be held. To secure the pillar from injury, a mud wall may be built round it, or a mud pillar over it, or both. To protect the wooden peg from injury by dry rot or white-ants, it should be soaked for four days in a solution of 1 lb. of sulphate of copper (*nila tutiya*) to 4 gallons of water, and should be coated with tar.

11. As bench-marks are intended for future reference, they are perfectly useless unless such a description of them be given as will serve to identify them indubitably. In addition to a sketch, the position with reference to well defined and easily seen objects in the neighbourhood, and to the north point, the name of the village in whose lands it is situated; if a tomb, the name of the person buried there; if a well, names of owners and local appellation (if any), if a boundary mark (none but those at the intersections of more than two village boundaries should be taken), the names of the villages whose lands meet there; should all be noted in the field-book. The cornice of a building is a very safe place to have a Bench Mark and it can be given a value by holding the staff upside down. This method of holding a staff upside down on both sides of a wall is the best way to continue levelling across such an obstacle. The Surveyor of course choosing a good horizontal course or slab on the top where his line crosses. In the first case the fore staff is additive and in the second subtractive to obtain the H. I. of instrument in the further position.

12. All *observations* and information connected with either survey or levels should be entered in the field-book *at the time of noting in ink in the field.* Nothing should be trusted to memory.

13. The date of the survey, as well as the number and maker's name of the instrument in use, should never be omitted.

14. The north and south line should be drawn through the centre of a plan and be as long as possible. The magnetic meridian should not only be marked on the plan, but the amount of variation (and date of observation) also should appear on the face of the plan.

15. Field-book should be checked and reduced levels inked in at the conclusion of each day's work—the survey should invariably be plotted from the *original* field-book. If a fair copy is made it should be as a *duplicate* only in case of accident to the original, but any surveyor making a fair copy to be plotted from it on the pretence that the original is too dirty to be sent in, should meet with no mercy.

16. Field-books should be properly indexed and work should have as many cross references as are necessary, and be recorded in ink.

17. All erasures and corrections in field books must be made in ink and verified by initialling and date.

95. **Demarcation.**—Although hardly partaking of the nature of a regular survey operation, the system described below, of recording in a graphic manner the relative contour of the surface of the ground and other details, is well worthy of the attention of surveyors engaged in irrigation or drainage works in cultivated tracts of the plains of India.

The system is only applicable to cultivated tracts, as it is a graphic systematic record of the knowledge of the cultivators of the ground, modified and possibly corrected now and then by the experience of the surveyor or observer. The object with which demarcation observations are started in any tract is to prepare on a large scale, a map or chart showing the following :—

(i) The exact lines along which drainage flows off the fields, and eventually finds its way to the rivers on either side.

(ii) The exact positions of the watersheds dividing these drainages.

(iii) The areas and distribution of the main qualities of soils.

(iv) The areas and positions of the "~~command~~" of wells or other sources of irrigation which it is desired to preserve or prevent interference with.

(v) The actual position of level pegs, or other survey marks, and of the land occupied by works about to be constructed.

It will be evident to all concerned in such operations that if the above-mentioned minute information can be cheaply and accurately acquired the calculations of discharge off catchment areas, discharge required to irrigate certain lands, or to pass down distributaries and canals, will be rendered simple and precise, and that it will be possible to carry out the alignments of channels with more accuracy and certainty than is likely to be the case with the most detailed contour survey founded on lines of spirit levelling.

The procedure is simple enough. Two copies are made of the village map of each village contained within the area to be demarcated : both copies are on cloth, one on a separate sheet and the other on a large sheet, joined up at the boundaries with copies of the maps of all villages within the tract. It may be mentioned that when large tracts of country are dealt with in order to keep the large sheets of convenient size, it will be necessary to trace the maps according to minor doabs.

These maps are generally printed on a scale of 16 inches to the mile or thereabouts, and show the field boundaries and numbers, village sites, roads and tanks, and other important features. After being copied on

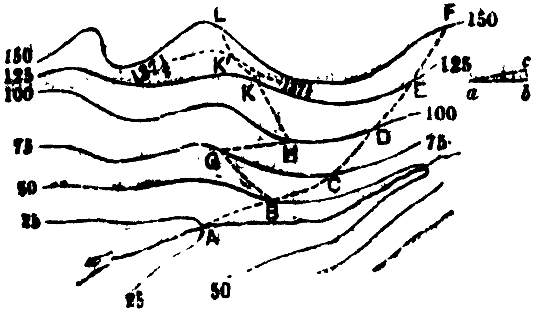
cloth these features should be suitably colored up on both copies, and then from the written village record the various qualities of soil can also be colored up, as the numbers of the fields given in the village map and records are the same there is no difficulty in doing this, indeed all through, the map should be looked on as a graphic index to the records. After soils, the irrigation of wells and other sources can be shown on the map from the village season or other records, the results of one or a series of years being taken according to the degree of accuracy aimed at. Regarding the record of possible irrigation from, or command of wells and other sources of supply, it may be noted that very few sources except good wells and canals can be considered permanent, and that the record on the map of a canal project of any existing irrigation but that from good wells, is hardly necessary. To obtain the proper command of a well the results of three years' working at least should be plotted, and a thick boundary line should be drawn round this area, including any dry fields surrounded by irrigated fields.

It will save time to plot all the above information on the separate maps, as a large number of men can then be employed extracting and plotting figures, it only takes a short time to trace the areas once plotted on to the large sheet.

The small sheets are now ready for out-door work. The surveyor, therefore, provided with the village map, should visit a village, and having enlisted as his guides a few respectable cultivators, should enquire the direction of flow of rain water of the fields he is standing on; noting the reply, he should walk on until he finds a point from which the water divides or flows in opposite directions—this is clearly a point on a watershed, and should be marked on the map with a pencil thus, X. In the same manner the points on drainages can be determined and marked with arrows, > *viz.*, the points where the rain water from two or more fields meet, and flow off together. When a few points on watersheds or drainages are once fixed it is easy enough to walk along and mark the lines, being careful to follow the information given by the cultivators in preference to judging by the eye. By the time most of the village area has been traversed the map will be ready to have the junctions of watersheds and drainages filled in, and this should be done with great care and after due local enquiry.

96. **To lay out the direction of road on a hill-side at any given slope.**

Fig. 28.



Scale—8 inches to 1 mile.

Let the figure represent the contoured plan of a hill-side along which it is required to lay out the direction of a road, rising from the point A to the level 150 at a uniform gradient of 1 in 8. As the interval between the contours is 25 feet, the plan of the road between any two contours will be the base of a right-angled triangle *abc*, in which $ab = 25 \times 8 = 200$ feet. Laying of this distance on the same scale as the map (8 inches to 1 mile) between the successive contours at BCD, etc., the required direction is determined. Should a zigzag not be objectionable, the road may be made to follow the course BGHKL.

It sometimes happens that the course of the road between two contours close together is not quite clear. In this case an intermediate contour must be interpolated, as 137½ in the figure, and the KK′ = 100 feet laid off between 125 and 137½, and a further length K′L of 100 feet between 137½ and 150.

As an exercise the student is recommended to the following example :—

Draw four concentric circles, ½-inch apart: diameter of the smallest circle ·75 inch. Assuming these circles to be the contours (50 feet interval) of a conical hill, draw the track of a road ascending from the lowest contour to the summit, at a slope of 1 in 20. Scale—6 inches to 1 mile.

97. **To find the boundary of excavation or filling on a contoured plan.**—In a problem of this nature, *see* fig. 29, it is necessary to assume that A and B are more or less ruling points or that A and B by trial have been found to give a better gradient, and avoid expensive bridging, protection of permanent way, etc.

To find the boundary of excavation it is necessary therefore first to lay down on the survey the road alignment with the adopted width of road-bed

and to show thereon the probable gradient. Next at right angles to the alignment the plan of the contours representing the slope of embanking or cutting—in this case $1\frac{1}{2}$ to 1. The points on the map where these contours intersect the contours of the surveyed map furnish the cutting edge of excavation and the filling edge required. Quantities can be worked out from the above with sufficient accuracy for a preliminary estimate.

98. **Setting out gradient.**—To set out a gradient it is easier to use a theodolite and having set it up fix on a rod the height of the axis of the telescope, then intersect this height above ground using the following rule. For gradients not steeper than 1 in 10 the angle representing that gradient will be found by the formula $\frac{3438}{n}$ minutes where n is the grade. Thus for a gradient of 1 in 60 the vertical angle would be $\frac{3438}{60} = 57{\cdot}3$ minutes.

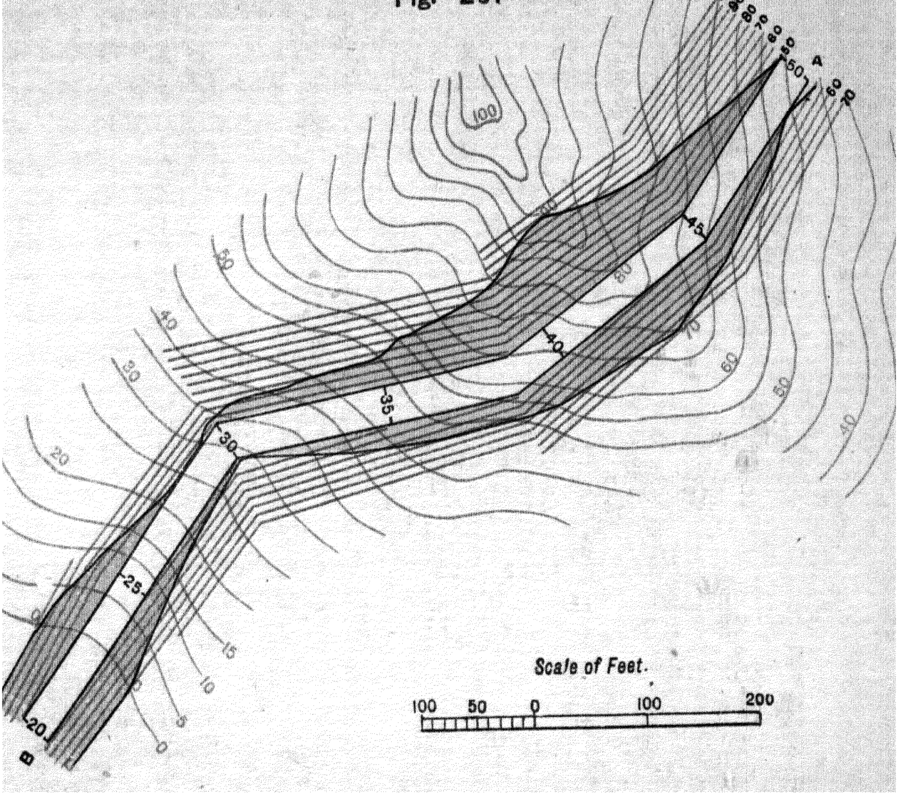
99. **Underground or mine surveying** for the following obvious reasons demands from the surveyor the most accurate work. The correct location above ground of the claim underground so that minerals, sometimes very valuable, may not be extracted in property belonging to another claim ; the correct location underground so that ventilating shafts, drainage problems, transportation, etc., can be efficiently dealt with ; the avoidance of old workings which when pierced might lead to serious accidents. Then again underground surveying means very often working in cramped situations, with station marks on the roof of the tunnel from which are suspended lamps or candles, when vertical readings are as important as horizontal ones, where chain or tape measures in darkness are made over slippery inclines, where work is scarcely ever closed with a check and where rapid work is essential. In fact mine surveying may be considered to be in a class of itself necessitating the use of theodolites fitted with auxiliary telescopes and special designs of compasses called dials, etc., and reference should be made to books specially written on the subject.

The chief difficulty in underground surveying is the transference of meridians from the surface to the shaft, and there are two cases involved. Case I. when the entrance to a mine is by a sloping tunnel or shallow shaft, and Case II. when the surface meridian has to be transferred down a vertical shaft.

Case I. presents no difficulty except in the instance of a tunnel when borings are being made at each end, and in order that they shall meet

CONTOUR PROBLEM IN ROAD AND RAILWAY CONSTRUCTION

Fig. 29.



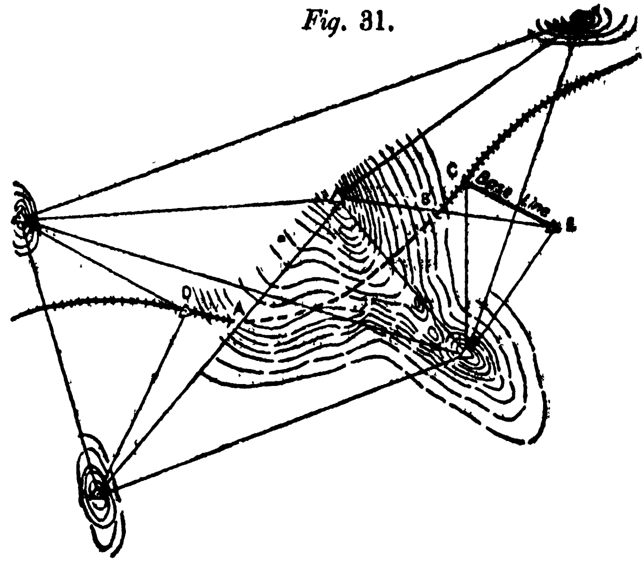
The above figure shows the plan of a Railway track through a hill with side slopes $1\frac{1}{2}$ to 1. Gradient of line 1 in 30. The portion shown red is the plan of the cutting required, and the portion in blue that of the Embankment

Note.—When road has been cut through hill the contour lines passing through road in plan will disappear and follow the blue lines along the side slopes

Photo. Zinco. August, 1926.—No. 420

Fig. 31 is a sketch of a piece of triangulation carried out to

Fig. 31.



determine accurately the directions DA and CB and also the horizontal distance between A and B. The height of B with respect to A must be accurately determined by levelling to obtain the required gradient, and when the gradient has been settled on, the distance along the curve is next obtained, and co-ordinates to points on the curve at certain distances can be worked out. The positions of ventilating shafts on the hill side can be fixed by angular measurements from any of the triangulated stations and involve problems where angles, etc., are deduced from the coordinates of stations and those of positions on the alignment where the shaft is required.

CHAPTER V.

HYDRO-ELECTRIC POWER SURVEYS.*

100. **Introduction.**—Electricity derived from water-power or white fuel as it is some times called but more generally known as hydro-electric is a branch of engineering which embraces Civil, Mechanical and Electrical.

The civil engineer, one might say, stops at the power house or generating station where the mechanical and electrical engineer steps in takes control of the machinery such as turbines, Pelton wheels, dynamos transformers and the ~~wiring of the~~ transmission line. The civil engineer thus delivers water full of potential energy which the electrical and mechanical adapts and puts to different uses in the form of power.

We are concerned mainly with the civil engineer's part of the project or scheme but it is as well to consider first the commercial aspect of the undertaking.

At present the greater percantage of electrical power is derived through engines driven by steam or gas and it may be accepted as a truism, other things being equal, that steam or gas have an advantage over water gravity power when the plant for the latter is not fully loaded or when the load factor is bad, while on a good load factor water power has an immense advantage specially if coal is dear at the locality where power is required. Those who wish to go further into this matter are referred to pages 22 to 25 of the Preliminary Report on Water Power Resourses of India. (Government of India) 1919.

101. **Water Power Schemes.**—The requirement is, for the engineer to devise some scheme by which water can be led from one level to another lower level so that in volume and head sufficient potential energy is generated on a turbine to make it a profitable undertaking; thus the first essential is water and for a good load factor the supply should be constant, next a head, the higher, the better, though it will be shown later that they are interdependent.

---

* Since this was written as lecture notes for students without going deeply into the subject the Triennial Report Hydro Electric Survey of India by J. W. Meares, C.I.E., F.R.A.S, M.I.C.E. has been published and reference should be made to this excellent publication also "Electrical Engineering Practice" by the same author.

Water Sources and Supply.—Water derived from a perennial river snow or spring fed must be considered the best source of supply. Water may be collected in a lake or tank and be guided into a channel by means of weirs and regulators and where the supply is insufficient, for say 3 to 4 dry months of the year, a combination of river and storage reservoir is then resorted to.

As regards the head there may exist a natural fall *in situ* or by a slight diversion a channel be made to serve the purpose, as at Niagara, Cauvery, etc., and possibly in the near future at Girsoppa.

If nature does not in one sudden drop supply the requisite head the engineer has then to consider how he can divert the water through open or closed channels along or through a hillside or spur till he obtains the head he requires. If he is able to divert the water across a watershed he often obtains a greater fall. The Tata schemes in the Bombay Presidency are notable examples of this principle and water which would ordinarily flow into the Bay of Bengal falls down the Western Ghats into the Arabian Sea. Take for instance the proposed Koina Valley project. Water which flows now into the Krishna river will in the near future be diverted through a tunnel into the Konkhan.

The now famous Periar Lake is a fine example to study but this is, at present, only used for irrigating land on the East coast and has not been harnessed for power. This so called diversion of water from one watershed to another generally entails tunnelling, usually a costly item, and yet on very large schemes will often be the cheapest method. It must however be borne in mind that there are vested rights in water. "Peter cannot be robbed to pay Paul" without causing friction and the probable abandonment of an otherwise splendid project.

Thus a little thought will show that an ideal scheme is one with a large perennial supply or constant supply with a high head and the worst scheme where large expensive works have to be carried out and maintained to impound water and then divert it a considerable distance to a low head. The location of the scheme with reference to the market it is to cater for, and the distance from such, is also a commercial proposition, as the further away the location, the greater the cost in transmission. The power loss in transmission is a factor of the greatest importance. Finally the use of the tail escape water for irrigation may be an additional source of income.

Q. × H.—A navigable river with a large discharge and a gentle slope of bed is of no use in such cases as the fall will be practically *nil* and though the product of discharge and head divided by some constant

gives the power generated yet the advantages of 5 cusecs falling a 1,000 feet is evident as against 1,000 cusecs falling 5 feet. The more profitable schemes are likely to be therefore those of small discharge and high head.

50 to 200 cusecs falling 400 to 2,000 feet may be classed as the best, though medium discharges of say 200 to 500 cusecs falling 100 to 400 feet are also good. "Large discharges falling from 5 to 50 feet are expensive in construction and are therefore poor if anything better is available. Any site is worth considering where the product of the minimum discharge in cusecs multiplied by the height of the fall in feet is not less than 1,600′. Anything smaller than this is too small for industrial purposes though it may be suitable for a town lighting supply."

• 102. **Preliminary Reconnaissance.** *Instruments Required.*— 1 Aneroid barometer. 2 Watch with a seconds hand. 3 Tape measure or 10′ rod. 4 An Abney~~'s Level~~.

The engineer will be required to give the following :—approximate minimum discharge, head available, position of site on river and reference number of map, accessibility of site, that is, nearest road, railway or steamer ghat; general remarks as to height of maximum flood, whether storage appears possible; nature of country, geological formation, any difficulties; market for power, materials for construction, upkeep of works and any other notes he may consider worthy of mention.

For a discharge as a preliminary, mark off 100′ on a straight bit of river of which you have a rough cross section and according to Barlow using floats (bottles half filled with water are very efficient) and taking surface velocity you obtain $Q = \frac{\text{velocity} \times \text{wetted sectional area}}{2}$ or a river 40′ wide $1\frac{1}{2}'$ deep, average, with a surface velocity of 2′ a second = 60 cusecs. 10 or 12 floats may be sent down the stream in different positions of its width and timed along the 100 feet length. Here the seconds watch is necessary and better still a stop watch. The Aneroid barometer will give (after correction) heights above mean sea level though no great accuracy can be obtained; it is rather the difference in heights between two places which is best determined. The Abney level that will give a good idea of the 0 contour level along site or possibly the height the impounded water will reach, if a dam is necessary.

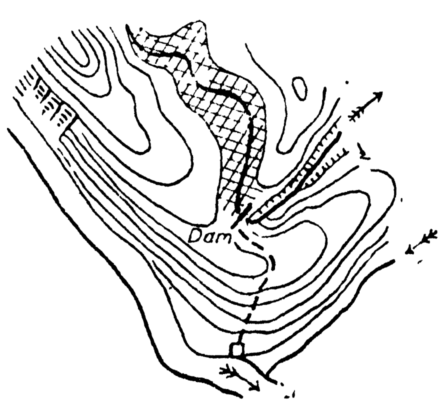
A light pattern plane table with ordinary sight rule and magnetic compass if carried would be a most useful adjunct in the hands of a man who can use it with any confidence, as with it, a great deal more information could be plotted on the spot and a very fair alignment marked on the map, supplemented by a connecting chain of heights obtained by the Abney corrected for curvature and refraction. Natural scale of tangents might be carried in a note book.

103. Study of Maps.—In the study of maps the following hints will be found useful remembering that the larger the scale the more exact the information also the more recent the survey of the map the more reliable the detail drawn such as roads, paths, limits of cultivation, forests, and grass lands. It must always be borne in mind, especially in hilly tracts, that villages are often evacuated or moved a short distance away and a map is some times condemned as inaccurate because it does not appear true. A map must be judged by its permanent features.

The presence of villages, patches of cultivation, is a good guide as to whether the locality is thinly or densely populated. If the map is on a scale of 1″ = 1 mile, or even one on a smaller scale, no great reliance should be placed on individual contours which are eye contours only. They are a good guide but nothing more. Here and there a few heights are taken on these maps and interpolation is resorted to. However in studying such a map it will be seen that contours open out in places. This indicates a spur or a plateau or a fairly level piece of ground and where the contours are close it indicates a steep piece of ground almost a cliff or bluff. Where contours at intervals leave a stream bed and ultimately merge into a hillside it shows that the stream is of gentle slope but on the other hand if contours are bunched together at the stream it means that there are cascades if not waterfalls existing. When a stream has a tortuous course or meanders it shows that its bed slope is small and such localities are not usually of any use for water power schemes except as sites for impounded water and here again they will be found useless if the stream runs in a deep gorge as then the dam would have to be a high one to obtain a sufficient water spread.

If on the other hand a site is found where a perennial stream approaches a river and then suddenly veers away and after a tortuous course enters the river close by (*see* Fig. 32) then a dam and a tunnel may prove a possible location for a power scheme. Such positions should always be examined as they are the next best to those in which the water is diverted from one watershed to another. Another case which occurs is when a stream makes a hairpin bend that is, returns on itself almost but at a much lower level. There is such an example on the Jumna river where it is possible to tunnel across the gap and obtain the requisite fall.

Fig. 32.



Scale 1″ = 1 mile. Contours 100′ interval.

The map will be helpful to mark off the catchment area and local authorities should be able to supply information as to extent and duration of rainfall but unless rain gauges have been systematically recorded no great reliance can be placed on the extent of run off but the highest limit of floods, and an investigation of these forms part of the reconnaissance work.

In the latest Survey of India maps contours are shown in *brown* streams in *black* except where they are perennial, that is never known to run dry, when they are shown in blue ; villages, roads of every description, bridges and all masonry structures in *red* ; railways, boundaries and symbols in *black* ; wooded areas in green ; and cultivation in yellow. For the scale 1″ = 1 mile these official maps may be taken to be very accurate and sufficient for all purposes of reconnaissance.

104. The following memory notes will be found useful.

1000 mill : c. f. = 30 cusecs a year continuous discharge allowing for absorption and evaporation.

1 square mile of water 1 foot deep will give $\frac{7}{8}$ cusecs discharge for 12 months or a rough rule of 1 cusec for 12 months.

1 inch of rain = 100 British Metric tons or 3,640 c. f., per acre ∴ 64,000 tons to the square mile or weighs 100 British Metric tons to an acre. (One acre = 10 × 1 Gunter's chains).

Flow of 10 cusecs perennial = 11·2′ depth with a surface of 1 square mile.

1 cusec flowing for 12 hours = 1 acre foot of water.

1 kilowatt = $\frac{4}{3}$ h. p. = 737 feet lbs. per second.

1 bigha = $\frac{5}{8}$ acre.

Current at 0·1 anna a unit (*i.e.* 1 kw : hour) costs Rs. 55 per Kw. year.

Load factor is the ratio of the *average* supply of power to the normal maximum of the generator.

$$\frac{\text{Cusecs} \times \text{head}}{11} = \text{electrical horse power.}$$

$$\frac{\text{Cusecs} \times \text{head}}{15} = \text{kilowatts.}$$

$$\frac{\text{Cusecs} \times \text{head}}{20} = \text{kilowatts as delivered.}$$

See Electrical Engineering Practice by J. W. Meares, para. 346.

105. **Rainfall and Run off.**—In India (excluding all snow fed rivers), streams, rivers, and tanks derive their water mostly from the S. W. monsoon concentrated between the months of June and October though

some parts of the Peninsular obtain rain during December and January such as Madras and the S. E. coast and a few falls may occur under the influence of the S. E. monsoon all over India and in Northern India a few falls of rain in winter from the direction of Persia. This latter rainfall does not give much run off as it occurs after a spell or break of dry weather. It is however not a negligible quantity and at times it has often more than compensated for evaporation which has taken place in the interim. The engineer will at any rate expect his maximum floods to occur during the regular monsoon period and he must allow for this discharge in his dam or weir or whatever arrangements he has made for escaping the overflow. No hard and fast rules can be laid down for quantity of Run-off. Each locality requires its own formula and should be examined on its own merits. Soil forests, lie or land, prevailing winds, duration of monsoon, average rainfall, etc. must be considered and should be examined on the spot and local knowledge gained as to maximum known height of floods on rivers or streams.

Sir Alexander Binnie lays down for the Central Provinces 40% of average rainfall after deducting 20 to 25% to allow for 3 successive dry years. This has been found to be a fairly liberal estimate and one which would be classed as high for say Bundelkand. For the Mirzapur water supply scheme 25% of average rainfall was agreed upon.

The engineer can not leave anything to chance and therefore must work on a minimum run-off as well as a maximum discharge for his escape. Example—take the average rainfall of 60" over a certain area of the Central Provinces of 20 sq. miles. If we reduce this by 25% we obtain 45" and 40% of this is 18" or $1\frac{1}{2}'$ of water. If all this were impounded it gives us 30 sq. mile feet as a result and since $\frac{7}{8}$ cusecs discharge is equal to 1 sq. mile foot of water we obtain a minimum discharge of $26\frac{1}{4}$ cusecs. Now it is not possible to impound all this water, a good deal may come down in a sudden rush and be lost in discharge through the escape, further a tank is never drained dry so that $26\frac{1}{4}$ cusec may ultimately be reduced to 15. Again 30 sq. mile feet of water means a tank with a surface area of 1 sq. mile and 30′ average depth and so it must be decided whether the dam can be raised to a height of at least 50′ to 60′ to impound this. When the engineer does so he must next consider the cost of such a dam and thus it will be seen that one item leads to another ending in not so much as to whether there will be sufficient water but whether in the end it is a sound commercial proposition and so on.

106. **Pipe Line.**—The next item to consider is the pipe line or channel to the forebay or balancing tank, and it is an axiom in hydro-

electric schemes to lose as little head as possible. An examination of the formula $\frac{\text{cusecs} \times \text{head}}{20}$ will show that 10′ of head lost between the dam site and forebay means a great deal of power lost and lost irretrievably. The forebay or tank reservoir is constructed to gather silt and by a system of strainers to catch all matter likely to damage the turbines or Pelton wheels.

107. **Pressure Pipes.**—Modern practice is to give each turbine a separate pipe. If the maximum capacity of discharge is divided by 3 there will be three units but an extra unit is always put in to allow for a breakdown so that in the above, 4 pipes would be taken to the power house. This system always enables one to regulate the load capacity. There is also one other limit to be considered or imposed for the pipe line. The thickness of metal cannot be much over 1¼″ for rivetted pipes or even for welded pipes, in fact 1⅛″ is fairly high. Knowing the head you can calculate the maximum diameter that you can go to, and this gives you the maximum quantity of water that any one pipe will deliver. This again determines your size of turbine and from the capacity and speed of each unit you can readily get quotations for generators, switch gear, transformers, etc.

In pipe lines a speed of 6 to 10′ a sec can be allowed in the pipe but this must be checked against friction loss resulting. Another factor is water hammer action, to counteract which a water tower is necessary or a surge chamber. If the length of the pipe line is more than 5 times the head, water hammer is inevitable. The pipe line has to be bracketted to the rock face and the actual length of the pipe down the scarp is an important detail and not an easy matter for survey. The manufacturer must know, besides the true length and the angle for all bends and the number and positions of expansion joints required.

Pipes will of course have to take larger stresses as they approach the bottom and the latest practice is to lessen the diametre and give a greater thickness of metal.

In the Andhra valley scheme the pipes are 42″ at the top then 36″ and then 32″ at the bottom.

108. **Tail Race.**—When the water has passed through the turbine it has still a certain amount of velocity and so that it will not surge at the back of the turbine it must be carried away quickly and then allowed to enter a channel doing as little damage as possible in its journey. If there are several tail races then there should be differences of level to prevent backing up or afflux. These channels are usually of masonry or concrete and are sometimes built under the transformer building to economise space.

109. **Transmission Line.**—Here the surveyor will be called to select the shortest and cheapest alignment and to calculate for and construct the pillars or grids for the line. Very often the alignment will be over hilly country and the use of the bar-subtense will prove almost invaluable for measuring distances across gorges. (Compare device on the India Pattern Level and the use of the graduations on the micrometer wheel).

Thus in this brief outline it is seen that the subject embraces almost every class and type of civil engineering, *viz*:—buildings, such as power house, transformer station, dams, weirs, canals, channels, acqueducts, suspension bridges and aqueducts to carry water across a valley, sluices reservoirs, forebays, tail escape, towers, etc. The student is recommended the following:—Buckley, Binnie and Strange for water supply and run-off, Wegmann and Strange for dams and Mears for Electrical Engineering generally.

We may now consider more fully certain questions of run off, capacity of tanks, evaporation and absorption. The methods of detail survey, can then be left to the engineer who should discriminate between the necessity of triangulation or traversing, with the plane table as a detail filler according to the accuracy needed.

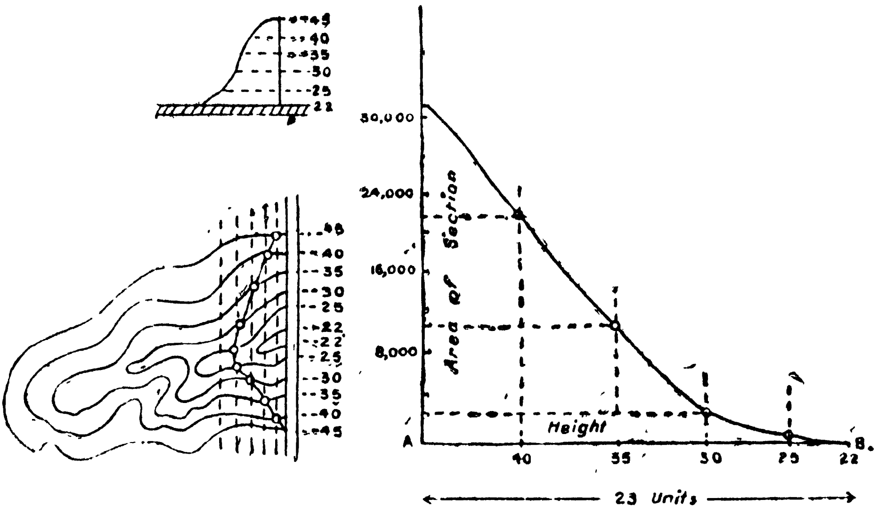
110. **Barlow's Percentages** are as follows :—

		Flat cultivated black cotton soil catchment.	Flat cultivated and stiff soils.	Average catchment.	Hills and plains with little cultivation.	Very hilly steep and rocky with little cultivation.	
½ to 1" in 24 hours if preceded or ended by showers of 1" to 1½" in 24 hours *otherwise*	LIGHT	1 %	3 %	5 %	10 %	15 %	Compare the Arthur Hill scheme where 230" of rain fall on barren bare rocky hill sides in about 3 months and where the run off to fill the tank is expected to be 50 % of total.
1" to 1½" if followed or preceded by rain also or 1½ to 3½ *otherwise*............	MEDIUM	10 %	15 %	20 %	25 %	33 %	
Those over 3" or continuous falls of 2" in a day or falls of the intensity of 2" in an hour or over	HEAVY	20 %	33 %	40 %	55 %	70 %	

111. The following is a graphic method of finding the capacity of a tank.

Example:—Find the volume of water in a reservoir when the water stands at a level of 45′ above datum ; the bottom of the reservoir being 22′.

Fig. 33.



First draw elevation and plan of dam and then take surface areas of each contour by planimeter.

Let area of 45 contour = 5·083 sq. inches and if linear scale is 80′ = 1″ then area = 5·083 × 80 × 80 = 32,500 sq. ft.

and areas of 40 contour		=	21,530
35	„	=	10,560
30	„	=	3,780
25	„	=	577
22	„	=	0

The length of the irregular solid = 45 – 22 = 23 therefore on a hor: scale of 1″ = 10′, 23′ = A B ; vertical scale so that 1″ = 16,000 sq. ft ; and thus 1 sq. in. on the paper represents 10 × 16,000 = 160,000 cubic feet.

Now area of paper = 1·638 sq. in. therefore vol. = 16,000 × 1·633 = 261,600 cubic feet = 1,634,000 galls.

This example is to be found in the "D. U. Series Vol. 1. Mathematics for Engineers" page 332.

TABLE I.

Values to be used for Curvature and Refraction in Height Computations, Form E, between latitudes 23° and 80°.

0′	Log feet.	0′	Log feet.	1′	Log feet.	1′	Log feet.
″		″		″		″	
0	0·000	31	3·860	0	4·147	31	4·328
1	2·369	32	·874	1	·154	32	·332
2	·670	33	·887	2	·161	33	·337
3	·846	34	·900	3	·168	34	·342
4	·971	35	·913	4	·175	35	·346
5	3·067	36	·925	5	·181	36	·351
6	·147	37	·937	6	·188	37	·355
7	·214	38	·948	7	·195	38	·360
8	·272	39	·960	8	·201	39	·364
2	·323	40	3·971	9	4·207	40	4·369
10	3·369	41	·981	10	·214	41	·373
11	·410	42	·992	11	·220	42	·377
12	·448	43	4·002	12	·226	43	·381
13	·482	44	·012	13	·232	44	·386
14	·515	45	·022	14	·238	45	·390
15	·545	46	·031	15	·244	46	·394
16	·573	·47	·041	16	·249	47	·398
17	·599	48	·050	17	·255	48	·402
18	·624	49	059	18	·261	49	·406
19	·647	50	·067	19	·266	50	·410
20	3·670	51	·076	20	4·272	51	4·414
21	·691	52	·085	21	·277	52	·418
22	·711	53	·093	22	·282	53	·422
23	·730	54	·101	23	·288	54	·425
24	·749	55	·109	24	·293	55	·429
25	·766	56	·117	25	·298	56	·433
26	·783	57	·124	26	·303	57	·437
27	·800	58	·132	27	·308	58	·440
28	·816	59	·139	28	·313	59	·444
29	·831	60	·147	29	·318	60	·448
30	3·846	61	·154	30	4·323	61	·451

Example.—Let log base = 4·1945328 and the observed vertical angle can be − 0° 17′ 19″. In the table the nearest number is 4·195 = + 1′ 7″ and therefore the corrected angle will be − 0° 17′ 19″ + 1′ 7″ = − 0° 16′ 12″.

TABLE II.

Correction for Curvature and Refraction.

Showing the difference of the Apparent and True Level in feet, and Decimal parts of Feet, for distances in Feet, Chains and Miles.

Distance in Feet.	CORRECTION IN FEET.			Distance in Chains.	CORRECTION IN FEET.			Distance in Miles.	CORRECTION IN FEET.		
	For Curvature.	For Refraction.	For Curvature and Refraction.		For Curvature.	For Refraction.	For Curvature and Refraction.		For Curvature.	For Refraction.	For Curvature and Refraction.
100	·00024	00004	00020	1·0	·00010	00001	00000	¼	·0417	·0060	·0357
150	·00054	·00008	.00046	1·5	·00024	·00003	·00021	½	·1668	·0238	·1430
200	·00096	·00018	00083	2·0	00042	00006	·00036	¾	·3752	·0536	·3216
250	·00149	·00021	00128	2·5	00065	·00009	·00056	1	6680	.0953	·5717
300	·00215	·00031	·00184	3·0	·00094	·00013	·00081	1½	1 5008	.2144	1·2864
350	·00293	·00042	00251	3·5	·00128	00018	·00110	2	2 6680	·3811	2·2869
400	·00383	·00055	·00328	4 0	00167	·00024	·00143	2½	4·1688	·5955	3·5733
450	.00484	·00069	·00415	4·5	·00211	·00030	·00181	3	6 0030	·8561	5.1469
500	·00598	00085	00513	5 0	00261	00037	00224	3½	8 1708	1 1673	7 0035
550	.00724	·00103	00621	5·5	·00315	·00045	00270	4	10 6720	1·5246	9 1474
600	·00861	·00123	·00738	6 0	00375	·00054	00321	4½	13 5468	1·9295	11.5773
650	·01010	·00144	·00866	6·5	00440	·00063	·00377	5	16 6750	2 3821	14.2029
700	·01172	00167	01005	7·0	00511	·00073	00438	5½	20 1769	2·8824	17.2945
750	·01345	·00192	·01153	7 5	·00586	00084	00502	6	24 0120	3·4303	20 5817
800	·01531	·00219	01312	8·0	.00667	·00095	·00572	6½	28 1800	4·0258	24.1551
850	.01728	·00247	01481	8·5	.00753	·00108	00645	7	32 6830	4·6690	28 0143
900	.01938	·00277	01661	9·0	.00844	·00121	·00723	7½	37·5190	5·3599	32.1591
950	.02159	00308	·01851	9 5	.00940	·00134	00806	8	42 6880	6·0997	56·5884
1000	.02392	00333	02059	10 0	.01042	·00149	00893	8½	48 1910	6 8844	41·3066
1050	.02638	00377	·02261	10·5	.01149	00164	00985	9	54 0270	7·7181	46 3089
1100	.02895	00414	02481	11 0	.01261	·00180	·01081	9½	60·1971	8·5996	51·5975
1150	.03164	00452	·02712	11·5	.01378	·00197	01181	10	66 7000	9·5286	57·1714
1200	03445	00492	.02953	12 0	.01501	·00214	·01287	11	80·7070	11 5296	69·1774
1250	·03738	·00534	.03204	12·5	.01628	00233	·01395	12	96 0480	13 7211	82 3269
1300	·04043	·00578	.03465	13 0	.01761	00252	·01509	13	112·7230	16·1033	96 6197
1350	.04361	00623	.03738	13·5	01899	00272	01628	14	130.7320	18·6760	112 0560
1400	.04689	00670	04019	14·0	.02043	00292	.01751	15	150 0750	21·4393	128 6357
1450	·05030	00719	.04311	14·5	·02191	00313	.01878	16	170·7520	24·3931	146 3589
1500	·05383	·00769	.04614	15 0	·02345	00345	.02010	17	192 7630	27·5376	165·2254
1550	·05748	·00821	.04927	15·5	02504	·00358	02146	18	216 1086	30 8727	185·2359
1600	06125	00875	.05250	16 0	02668	.00381	02287	19	246 7870	34·3981	206·2889
1650	·06514	·00931	.05583	16 5	·02837	.00405	02432	20	266 8000	38 1143	228 6857
1700	·06914	·00988	05926	17·0	03012	00430	02582				
1750	·07327	·01047	.06280	17·5	·03192	.00456	.02736				
1800	·07752	·01107	.06645	18·0	·03377	.00482	02895				
1850	·08188	·01170	07018	18·5	·03567	.00509	.03058				
1900	·08637	·01234	.07403	19·0	·03762	00537	·03225				
1950	·09098	·01300	.07798	19·5	·03963	·00566	·03397				
2000	·09570	·01367	.08203	20·0	·04169	·00596	·03573				

TABLE III.*—Astronomical Refractions.†

Apparent Zenith Distance.	Mean Refraction for Baro. 30 in. and Temp. 50° Fahrenheit.	Diff. for 1′ Z. D.	Diff. for 1 inch Barometer.	Diff. for 1° Temp.
°	′ ″	″	″	″
0	0 0·0	+ 0·017	0·00	0·000
1	1·0	·017	·03	·002
2	2·0	·017	·07	·004
3	3·1	·017	·11	·006
4	4·1	·017	·14	·008
5	5·1	·017	·17	·010
6	6·1	·017	·21	·012
7	7·2	·017	·24	·014
8	8·2	·017	·28	·016
9	9·2	·017	·31	·018
10	10·3	·017	·35	·021
11	0 11·3	0·017	0·38	0·023
12	12·4	·018	·42	·025
13	13·5	·018	·46	·027
14	14·5	·018	·49	·029
15	15·6	·018	·53	·031
16	16·7	·018	·57	·033
17	17·8	·018	·60	·036
18	18·9	·019	·64	·038
19	20·1	·019	·68	·040
20	21·2	·019	·72	·042
21	0 22·4	0·019	0·76	0·045
22	23·6	·019	·80	·047
23	24·7	·020	·84	·049
24	26·0	·020	·88	·052
25	27·2	·020	·92	·054
26	28·4	·021	·96	·057
27	29·7	·032	1·01	·059
28	31·0	·022	·05	·062
29	32·3	·023	·09	·065
30	33·7	·023	·14	·067
31	0 35·0	0·023	1·19	0·070
32	36·4	·023	·23	·073
33	37·8	·024	·28	·076
34	39·3	·025	·33	·079
35	40·8	·025	·38	·082
36	42·3	·026	·43	·085
37	43·9	·027	·49	·088
38	45·5	·028	·54	·091
39	47·2	·028	·60	·094
40	48·9	·028	·66	·098
41	0 50·6	0·029	1·72	0·101
42	52·4	·031	·78	·105
43	54·3	·032	·84	·109
44	56·2	·033	·91	·112
45	58·2	·034	·97	·116

Apparent Zenith Distance.	Mean Refraction for Baro. 30 in. and Temp. 50° Fahrenheit.	Diff. for 1′ Z. D.	Diff. for 1 inch Barometer.	Diff. for 1° Temp.
° ′	′ ″	″	″	″
46	1 0·3	+ 0·035	2·04	0·121
47	1 2·4	·037	·12	·125
48	1 4·7	·038	·19	·129
49	1 7·0	·039	·27	·134
50	1 9·4	·041	·35	·139
51	1 11·9	·042	·44	·144
52	1 14·5	·044	·53	·149
53	1 17·2	·047	·62	·154
54	1 20·1	·049	·72	·160
55	1 23·1	·051	·82	·166
56	1 26·2	0·054	2·92	0·172
57	1 29·5	·057	3·03	·179
58	1 33·0	·060	·15	·186
59	1 36·7	·064	·28	·193
60	1 40·6	·668	·41	·201
61	1 44·8	·072	·55	·210
62	1 49·2	·076	·70	·218
63	1 53·9	·081	·86	·228
64	1 58·9	·088	4·03	·238
65	2 4·4	·095	·22	·249
66	2 10·2	0·101	4·41	·260
67	2 16·5	·109	·63	·273
68	2 23·3	·118	·86	·287
69	2 30·7	·129	5·11	·301
70	2 38·8	·142	·38	·318
71	2 47·7	·155	·68	·335
72 0	2 57·4	·171	6·01	·355
30	3 2·8	·182	·20	·366
73 0	3 8·3	·190	·38	·377
30	3 14·2	·204	·58	·388
74 0	3 20·5	·215	·80	·401
30	3 27·1	·227	7·02	·414
75 0	3 34·1	0·237	7·26	0·428
10	3 36·5	·245	·34	·433
20	3 39·0	·253	·42	·438
30	3 41·6	·257	·51	·443
40	3 44·2	·263	·60	·448
50	3 46·8	·268	·69	·454
76 0	3 49·6	·275	·78	·459
10	3 52·3	·280	·87	·465
20	3 55·2	·287	·97	·470
30	3 58·1	·295	8·07	·476
40	4 1·0	·300	·17	·482
50	4 4·1	·310	·27	·488

NOTES.—1. Refraction is *additive* to Apparent Zenith Distance and *subtractive* from Apparent Altitude.

2. Correction for barometer is $\frac{\textit{subtractive from}}{\textit{additive to}}$ Mean Refraction if barometer reads $\frac{\textit{less}}{\textit{more}}$ than 30 inches

3. Correction for temperature is $\frac{\textit{subtractive from}}{\textit{additive to}}$ " temperature is $\frac{\textit{higher}}{\textit{lower}}$ than 50° Fahr

* Auxiliary Table XXII., Survey of India.

† 58″ cot elevation or altitude gives a close result suitable for India.

TABLE III.* —Astronomical Refractions.—(*Continued*).

Apparent Zenith Distance.		Mean Refraction for Baro 30 in. and Temp. 50° Fahrenheit.		Diff. for 1′ Z D.	Diff for 1 inch Barometer.	Diff. for 1° Temp.
°	′	′	″	″	″	″
77	0	4	7·2	+ 1·315	8 40	0 494
	10	4	10·4	·320	·51	·526
	20	4	13 6	330	62	·532
	30	4	17 0	·340	·74	540
	40	4	20·4	·345	85	547
	50	4	23 9	·355	97	·554
78	0	4	27 5	·365	9 09	52
	10	4	31 2	·375	·22	·570
	20	4	35 0	·385	·35	·578
	30	4	38 9	·395	·48	·586
	40	4	42·9	·405	·62	·594
	50	4	47·0	·415	·76	·603
79	0	4	51·2	0·430	9 90	0 612
	10	4	55·6	·440	10 05	·621
	20	5	0 0	·450	·20	·630
	30	5	4·6	·465	36	640
	40	5	9·3	·480	·52	·650
	50	5	14·2	·495	·68	·660
80	0	5	19 2	·505	10·85	670
	10	5	24·3	·520	11 03	681
	20	5	29 6	·540	·21	692
	30	5	35·1	·560	·39	704
	40	5	40 8	·575	·62	·716
	50	5	46·6	·590	82	·728
81	0	5	52·6	0·610	12 02	0 740
	10	5	58 8	·635	·24	·753
	20	6	5·3	·655	·46	·767
	30	6	11 9	675	68	·781
	40	6	18 8	·700	·92	·795
	50	6	25 9	·725	13·16	·810
82	0	6	33 3	·755	13 41	·826
	10	6	41 0	·780	·67	·842
	20	6	48 9	·810	·94	·858
	30	6	57 2	840	14·23	976
	40	7	5 7	·870	·56	·894
	50	7	14·6	9.10	·86	·913
83	0	7	23 9	0·945	15·18	0 932
	10	7	33 5	·980	·51	·998
	20	7	43 5	1 025	·85	1 020
	30	7	54 0	·070	16 21	·043
	40	8	4·9	·110	·58	067
	50	8	16 2	·160	·97	·092
84	0	8	28 1	1·215	17 38	1·118
	10	8	40 5	·265	·80	·145
	20	8	53·4	·325	18 24	·173

Apparent Zenith Distance.		Mean Refraction for Baro 30 in and Temp. 50° Fahrenheit.		Diff. for 1′ Z. D.	Diff. for 1 inch Barometer.	Diff. for 1° Temp.
°	′	′	″	″	″	″
84	30	9	7 0	+ 1 395	18 71	1·203
	40	9	21·3	·460	19 25	235
	50	9	36·2	·535	76	·268
85	0	9	52 0	1 620	20 31	1 302
	10	10	9	·70	·9	·34
	20	10	26	79	21·5	·44
	30	19	44	·89	22 1	·48
	40	11	4	·99	9	·53
	50	11	24	2 11	23·6	·57
86	0	11	46	21	24 4	·62
	5	11	57	·29	7	65
	10	12	9	·36	25·1	68
	15	12	21	·43	6	·70
	20	12	33	·50	26 0	·73
	25	12	46	·58	·4	·76
86	30	12	59	2·66	26 9	1 87
	35	13	12	·75	27·5	90
	40	13	26	·84	28 0	·94
	45	13	41	92	·5	·97
	50	13	56	3 02	29 0	2 01
	55	14	11	13	·5	04
87	c	14	27	23	30·1	·08
	5	14	43	·33	·7	·12
	10	15	0	·45	31 2	·25
	15	15	18	·58	·8	·29
	20	15	36	·71	32 5	·34
	25	15	55	·83	33 1	39
87	30	16	14	3 96	33 8	2·44
	35	16	34	4·12	34·9	·49
	40	16	55	·29	35·6	·54
	45	17	17	·44	36 4	·59
	50	17	40	·60	37 2	76
	55	18	3	·79	38·0	82
88	0	18	28	4·99	9	·88
	5	18	53	5 19	39 8	95
	10	19	20	·40	40 7	3·02
	15	19	47	·63	41 7	·09
	20	20	16	·86	42·7	28
	25	20	46	6·11	43·7	·36
88	30	21	17	6·38	44 8	3·45
	35	21	50	67	46·4	·54
	40	22	24	·98	47 8	·76

NOTES.—1. Refraction is *additive* to Apparent Zenith Distance and *subtractive* from Apparent Altitude.

2. Correction for barometer is $\frac{\textit{subtractive from}}{\textit{additive to}}$ Mean Refraction if barometer reads $\frac{\textit{less}}{\textit{more}}$ than 30 inches.

3. Correction for temperature is $\frac{\textit{subtractive from}}{\textit{additive to}}$ " temperature is $\frac{\textit{higher}}{\textit{lower}}$ than 50° Fahr.

* Auxiliary Table XXII, Survey of India.

† 58″ cot elevation or altitude gives a close result suitable for India.

TABLE IV.—Sun's Parallax in Altitude.

Altitude.	Parallax.	Altitude.	Parallax.	Altitude.	Parallax.
	″	°	″	°	″
0	8·60	30	7·45	60	4·30
2	8·59	32	7·29	62	4·04
4	8·58	34	7·13	64	3·77
6	8·55	36	6·96	66	3·50
8	8·52	38	6·78	68	3·22
10	8·47	40	6·59	70	2·94
12	8·41	42	6·39	72	2·66
14	8·34	44	6·19	74	2·37
16	8·27	46	5·97	76	2·08
18	8·18	48	5·75	78	1·79
20	8·08	50	5·53	80	1·49
22	7·97	52	5·29	82	1·20
24	7·86	54	5·05	84	0·90
26	7·73	56	4·81	86	0·60
28	7·59	58	4·56	88	0·30
				90	0·00

TABLE V.*—Values of $\frac{2 \sin^2 \frac{1}{2} t}{\sin 1''}$ for Computation of Circumpolar Azimuths.

Seconds	Hour Angles in Time.									
	0m	1m	2m	3m	4m	5m	6m	7m	8m	9m
	"	"	"	"	"	"	"	"	"	"
0	0.00	1.96	7.85	17.67	31.42	49.09	70.68	96.20	125.65	159.02
1	0.00	2.03	7.98	17.87	31.68	49.41	71.07	96.66	126.17	159.61
2	0.00	2.10	8.12	18.07	31.94	49.74	71.47	97.12	126.70	160.20
3	0.00	2.16	8.25	18.27	32.20	50.07	71.86	97.58	127.22	160.80
4	0.01	2.23	8.39	18.47	32.47	50.40	72.26	98.04	117.75	161.39
5	0.01	2.31	8.52	18.67	32.74	50.73	72.66	98.50	128.28	161.98
6	0.02	2.38	8.66	18.87	33.01	51.07	73.06	98.97	128.81	162.58
7	0.02	2.45	8.80	19.07	33.27	51.40	73.46	99.43	129.34	163.17
8	0.03	2.52	8.94	19.28	33.54	51.74	73.86	99.90	129.87	163.77
9	0.04	2.60	9.08	19.48	33.81	52.07	74.26	100.37	130.40	164.37
10	0.05	2.67	9.22	19.69	34.09	52.41	74.66	100.74	130.94	164.97
11	0.06	2.75	9.36	19.90	34.36	52.75	75.06	101.31	131.47	165.57
12	0.08	2.83	9.50	20.11	34.64	53.09	75.47	101.78	132.01	166.17
13	0.09	2.91	9.64	20.32	34.91	53.43	75.88	102.25	132.55	166.77
14	0.11	2.99	9.79	20.53	35.19	53.77	76.29	102.72	133.09	167.37
15	0.12	3.07	9.94	20.74	35.46	54.11	76.69	103.20	133.63	167.97
16	0.14	3.15	10.09	20.95	35.74	54.46	77.10	103.67	134.17	168.58
17	0.16	3.23	10.24	21.16	36.02	54.80	77.51	104.15	134.71	169.19
18	0.18	3.32	10.39	21.38	36.30	55.15	77.93	104.61	135.25	169.80
19	0.20	3.40	10.54	21.60	36.58	55.50	78.34	105.10	135.80	170.41
20	0.22	3.49	10.69	21.82	36.87	55.84	78.75	195.58	136.34	171.02
21	0.24	3.58	10.84	22.03	37.15	56.19	79.16	106.06	136.88	171.63
22	0.26	3.67	11.00	22.25	37.44	56.55	79.58	106.55	137.43	172.24
23	0.28	3.76	11.15	22.47	37.72	56.90	80.00	107.03	137.98	172.85
24	0.31	3.85	11.31	22.70	38.01	57.25	80.42	107.51	138.53	173.47
25	0.34	3.94	11.47	22.92	38.30	57.60	80.84	107.99	139.08	174.08
26	0.37	4.03	11.63	23.14	38.59	57.96	81.26	108.48	139.63	174.70
27	0.40	4.12	11.79	23.37	38.88	58.32	81.68	108.97	140.18	175.32
28	0.43	4.22	11.95	23.60	39.17	58.68	82.10	109.46	140.74	175.94
29	0.46	4.32	12.11	23.82	39.46	59.03	82.52	108.95	141.29	176.56
30	0.49	4.42	12.27	24.05	39.76	59.40	82.95	110.44	141.85	177.18
31	0.52	4.52	12.43	24.28	40.05	59.75	83.38	110.93	142.40	177.80
32	0.56	4.62	12.60	24.51	40.35	60.11	83.81	111.93	142.96	178.43
33	0.59	4.72	12.76	24.74	40.65	60.47	84.23	111.92	143.52	179.05
34	0.63	4.82	12.93	24.98	40.95	60.84	84.66	112.41	144.08	179.68
35	0.67	4.92	13.10	25.21	41.25	61.20	85.09	112.90	144.04	180.30
36	0.71	5.03	13.27	25.45	41.55	61.57	85.52	113.40	145.20	180.93
37	0.75	5.13	13.44	25.68	41.85	61.94	85.95	113.90	145.76	181.56
38	0.79	5.24	13.62	25.92	42.15	62.31	86.39	114.40	146.33	182.19
39	0.83	5.34	13.79	26.16	42.45	62.68	86.82	114.90	146.89	182.82
40	0.87	5.45	13.96	26.40	42.76	63.05	87.26	115.40	147.46	183.46
41	0.91	5.56	14.13	26.64	43.06	63.42	87.70	115.90	148.03	184.09
42	0.96	5.67	14.31	26.88	43.37	63.79	88.14	116.40	148.60	184.72
43	1.01	5.78	14.49	27.12	43.68	64.16	88.57	116.90	149.17	185.35
44	1.06	5.90	14.67	27.37	43.99	64.54	89.01	117.41	149.74	185.99
45	1.10	6.01	14.85	27.61	44.30	67.91	89.45	117.92	150.31	186.63
46	1.15	6.13	15.03	27.86	44.61	65.29	89.89	118.43	150.88	187.27
47	1.20	6.24	15.21	28.10	44.92	65.67	90.33	118.94	151.45	187.91
48	1.26	6.36	15.39	28.35	45.24	66.05	90.78	119.45	152.03	188.55
49	1.31	6.48	15.57	28.60	45.55	66.43	91.23	119.96	152.61	189.19
50	1.36	6.60	15.76	28.85	45.87	66.81	91.68	120.47	153.19	189.83
51	1.42	6.72	15.95	29.10	46.18	67.19	92.12	120.98	153.77	190.47
52	1.48	6.84	16.14	29.36	46.50	67.58	92.57	121.49	154.35	191.12
53	1.53	6.96	16.32	29.61	46.82	67.96	93.02	122.01	154.93	191.76
54	1.59	7.09	16.51	29.86	47.14	68.35	93.47	122.53	155.51	192.41
55	1.65	7.21	16.70	30.12	47.46	68.73	93.92	123.05	156.09	193.06
56	1.71	7.34	16.89	30.38	47.79	69.12	94.38	123.57	156.67	193.71
57	1.77	7.46	17.08	30.64	48.11	69.51	94.83	124.09	157.25	194.36
58	1.83	7.60	17.28	30.90	48.43	69.90	95.29	124.61	157.84	195.01
59	1.89	7.72	17.47	31.16	48.76	70.29	95.74	125.13	158.43	195.66

* Auxiliary Table XXIII, Survey of India.

TABLE V.*—Values of $\frac{2\sin^2 \frac{1}{2} t}{\sin 1''}$ for Computation of Circumpolar Azimuths.

Seconds.	Hour Angles in Time.									
	10^m	11^m	12^m	13^m	14^m	15^m	16^m	17^m	18^m	19^m
	″	″	″	″	″	″	″	″	″	″
0	196.32	237.54	282.68	331.74	384.74	441.63	502.46	567.19	635.84	708.42
1	196.97	238.26	283.47	332.59	385.65	442.62	503.50	568.30	637.03	709.66
2	197.63	238.98	284.26	333.44	386.56	443.60	504.55	569.42	638.20	710.90
3	198.28	239.70	285.94	334.29	387.48	444.58	505.60	570.53	639.38	712.15
4	198.94	240.42	285.83	335.15	388.40	445.56	506.65	571.65	640.56	713.39
5	199.60	241.14	286.62	336.00	389.32	446.55	507.70	572.76	641.74	714.64
6	200.26	241.87	287.41	336.86	390.24	447.54	508.76	573.88	642.92	715.90
7	200.92	242.60	288.20	337.72	391.16	448.53	509.81	575.00	644.11	717.13
8	201.59	243.33	289.00	338.58	392.09	449.51	510.86	576.12	645.30	718.39
9	202.25	244.06	289.79	339.44	393.01	450.50	511.92	577.24	646.49	719.63
10	202.92	244.79	290.58	340.30	393.94	451.50	512.98	578.36	647.66	720.89
11	203.58	245.52	291.38	341.16	394.86	452.49	514.03	579.48	648.86	722.14
12	204.25	246.25	292.18	342.02	395.79	453.48	515.09	580.60	650.04	723.40
13	204.92	246.98	292.98	342.88	396.72	454.48	516.15	581.73	651.24	724.65
14	205.59	247.72	293.78	343.75	397.65	455.47	517.21	582.85	652.42	725.91
15	206.26	248.45	294.58	344.62	398.58	456.47	518.27	583.98	653.65	727.18
16	206.93	249.19	295.38	345.49	399.52	457.47	519.34	585.11	654.82	728.43
17	207.60	249.93	296.18	346.36	400.45	458.47	520.40	586.25	656.01	729.69
18	208.27	250.67	296.99	347.23	401.38	459.47	521.47	587.38	657.20	730.95
19	208.94	251.41	297.79	348.10	402.32	460.47	522.53	588.50	658.40	732.22
20	209.62	252.15	298.60	348.97	403.26	461.47	523.60	589.64	659.60	733.48
21	210.30	252.89	299.40	349.84	404.20	462.48	524.67	590.77	660.80	734.73
22	210.98	253.63	300.21	350.71	405.14	463.48	525.74	591.90	662.00	736.00
23	211.66	254.37	301.02	351.58	406.08	464.48	526.81	593.05	663.21	737.28
24	212.34	255.12	301.83	352.46	407.02	465.49	527.89	594.18	664.40	738.53
25	215.02	255.87	302.64	353.34	407.96	466.50	528.96	595.32	665.61	739.81
26	213.70	256.62	303.46	354.22	408.90	467.51	530.03	596.46	666.81	741.07
27	214.38	257.37	304.27	355.10	409.84	468.52	531.11	597.60	668.02	742.35
28	215.07	258.12	305.09	355.98	410.79	469.53	532.18	598.74	669.22	743.62
29	215.75	258.87	305.90	356.86	411.73	470.54	533.26	599.89	670.44	744.89
30	216.44	259.62	306.72	357.74	412.68	471.55	534.33	601.02	671.65	746.17
31	217.12	260.37	307.54	358.62	413.63	472.57	535.41	602.17	672.85	747.45
32	217.81	261.12	308.36	359.51	414.59	473.58	536.50	603.32	674.06	748.72
33	218.50	261.88	309.18	360.39	415.54	474.60	537.58	604.46	675.27	750.00
34	219.19	262.64	310.00	361.28	416.49	475.62	538.67	605.62	676.49	751.28
35	219.88	263.39	310.82	362.17	417.44	476.64	539.75	606.76	677.70	752.56
36	220.58	264.15	311.65	363.07	418.40	477.65	540.83	607.91	678.92	753.84
37	221.27	264.91	312.47	363.96	419.35	478.67	541.91	609.06	680.13	755.13
38	221.97	265.68	313.30	364.85	420.31	479.70	543.00	610.21	681.35	756.40
39	222.66	266.44	314.12	365.75	421.27	480.72	544.09	611.36	682.57	757.69
40	223.36	267.20	314.95	366.64	422.23	481.74	545.18	612.52	683.79	758.96
41	224.06	267.96	315.78	367.53	423.19	482.77	546.27	613.68	685.01	760.26
42	224.76	268.73	316.61	368.42	424.15	483.79	547.36	614.84	686.23	761.54
43	225.46	269.49	317.44	369.31	425.11	484.82	548.45	616.00	687.46	762.83
44	226.16	270.26	318.27	370.21	426.07	485.85	549.55	617.15	688.68	764.12
45	226.86	271.02	319.10	371.11	427.04	486.88	550.64	618.32	689.91	765.42
46	227.57	271.79	319.94	372.01	428.01	487.91	551.73	619.47	691.13	766.71
47	228.27	272.56	320.78	372.91	428.97	488.94	552.83	620.64	692.36	768.00
48	228.98	273.34	321.62	373.82	429.93	489.97	553.93	621.80	693.59	769.29
49	229.68	274.11	322.45	374.72	430.90	491.01	555.03	622.96	694.82	770.58
50	230.39	274.88	323.29	375.62	431.87	492.05	556.13	624.14	696.05	771.88
51	231.10	275.65	324.13	376.52	432.84	493.08	557.24	625.30	696.28	773.18
52	231.81	276.43	324.97	377.43	433.82	494.12	558.34	626.47	698.51	774.48
53	232.52	277.20	325.81	378.34	434.79	495.15	559.44	627.64	699.75	775.78
54	233.24	277.98	326.66	379.26	435.76	496.19	560.55	628.81	700.99	777.07
55	233.95	278.76	327.50	380.17	436.73	497.23	561.65	629.97	702.21	778.38
56	234.67	279.55	328.35	381.08	437.71	498.28	562.76	631.15	703.4	779.69
57	235.38	280.33	329.19	381.99	338.69	499.32	563.87	632.32	704.69	780.98
58	236.10	281.12	330.04	382.90	439.67	500.37	564.98	633.49	705.93	782.29
59	236.82	281.90	330.89	383.82	440.65	501.41	566.08	634.67	707.18	783.59

* Auxiliary Table XXIII., Survey of India.

TABLE VI.

For converting INTERVALS of SIDEREAL Time into Equivalent INTERVALS of MEAN SOLAR Time.

HOURS.		MINUTES.				SECONDS.			
Hours of Sidereal Time.	Equivalents in Mean Time.	Minutes of Sidereal Time.	Equivalents in Mean Time.	Minutes of Sidereal Time.	Equivalents in Mean Time	Seconds of Sidereal Time.	Equivalents in Mean Time.	Seconds of Sidereal Time.	Equivalents in Mean Time.
	h m s		m s		m s		s		s
1	0 59 50·1704	1	0 59·8362	31	30 54·9214	1	0·9973	31	30·9154
2	1 59 40·3409	2	1 59·6723	32	31 54·7576	2	1·9945	32	31·9126
3	2 59 30·5113	3	2 59·5085	33	32 54·5937	3	2·9918	33	32·9099
4	3 59 20·6818	4	3 59·3447	34	33 54·4299	4	3·9891	34	33·9072
5	4 59 10·8522	5	4 59·1809	35	34 54·2661	5	4·9864	35	34·9045
6	5 59 1·0226	6	5 59·0170	36	35 54·1023	6	5·9836	36	35·9017
7	6 58 51·1931	7	6 58·8532	37	36 53·9384	7	6·9809	37	36·8990
8	7 58 41·3635	8	7 58·6894	38	37 53·7746	8	7·9782	38	37·8963
9	8 58 31·5340	9	8 58·5256	39	38 53·6108	9	8·9754	39	38·8935
10	9 58 21·7044	10	9 58·3617	40	39 53·4470	10	9·9727	40	39·8908
11	10 58 11·8748	11	10 58·1979	41	40 53·2831	11	10·9700	41	40·8881
12	11 58 2·0453	12	11 58·0341	42	41 53·1193	12	11·9672	42	41·8853
13	12 57 52·2157	13	12 57·8703	43	42 52·9555	13	12·9645	43	42·8826
14	13 57 42·3862	14	13 57·7064	44	43 52·7917	14	13·9618	44	43·8799
15	14 57 32·5566	15	14 57·5426	45	44 52·6278	15	14·9591	45	44·8772
16	15 57 22·7270	16	15 57·3788	46	45 52·4640	16	15·9563	46	45·8744
17	16 57 12·8975	17	16 57·2150	47	46 52·3002	17	16·9536	47	46·8717
18	17 57 3·0679	18	17 57·0511	48	47 52·1364	18	17·9509	48	47·8690
19	18 56 53·2384	19	18 56·8873	49	48 51·9725	19	18·9481	49	48·8662
20	19 56 43·4088	20	19 56·7235	50	49 51·8087	20	19·9454	50	49·8635
21	20 56 33·5792	21	20 56·5597	51	50 51·6449	21	20·9427	51	50·8608
22	21 56 23·7497	22	21 56·3958	52	51 51·4810	22	21·9399	52	51·8580
23	22 56 13·9201	23	22 56·2320	53	52 51·3172	23	22·9372	53	52·8553
24	23 56 4·0906	24	23 56·0682	54	53 51·1534	24	23·9345	54	53·8516
		25	24 55·9044	55	54 50·9896	25	24·9318	55	54·8499
		26	25 55·7405	56	55 50·8257	26	25·9290	56	55·8471
		27	26 55·5767	57	56 50·6619	27	16·9263	57	56·8444
		28	27 55·4129	58	57 50·4981	28	27·9236	58	57·8417
		29	28 55·2490	58	58 50·3343	29	28·9208	59	58·8389
		30	29 55·0852	60	59 50·1704	30	29·9181	60	59·8362

TABLE VII.

For converting INTERVALS of MEAN SOLAR Time into Equivalent INTERVALS of SIDEREAL Time.

HOURS.				MINUTES.						SECONDS.			
Hours of Mean Time.	Equivalents in Sidereal Time.			Minutes of Mean Time.	Equivalents in Sidereal Time.		Minutes of Mean Time.	Equivalents in Sidereal Time.		Seconds of Mean Time.	Equivalents in Sidereal Time.	Seconds of Mean Time.	Equivalents in Sidereal Time.
	h	m	s		m	s		m	s		s		s
1	1	0	9·8565	1	1	0·1643	31	31	5·0925	1	1·0027	31	31·0849
2	2	0	19·7130	2	2	0·3286	32	32	5·2568	2	2·0055	32	32·0876
3	3	0	29·5694	3	3	0·4928	33	33	5·4211	3	3·0082	33	33·0904
4	4	0	39·4259	4	4	0·6571	34	34	5·5853	4	4·0110	34	34·0931
5	5	0	49·2824	5	5	0·8214	35	35	5·7496	5	5·0137	35	35·0958
6	6	0	59·1388	6	6	0·9857	36	36	5·9139	6	6·0164	36	36·0986
7	7	1	8·9953	7	7	1·1499	37	37	6·0782	7	7·0192	37	37·1013
8	8	1	18·8518	8	8	1·3142	38	38	6·2424	8	8·0219	38	38·1040
9	9	1	28·7083	9	9	1·4785	39	39	6·4067	9	9·0246	39	39·1068
10	10	1	38·5647	10	10	1·6428	40	40	6·5710	10	10·0274	40	40·1095
11	11	1	48·4212	11	11	1·8070	41	41	6·7353	11	11·0301	41	41·1123
12	12	1	58·2777	12	12	1·9713	42	42	6·8995	12	12·0329	42	42·1150
13	13	2	8·1342	13	13	2·1356	43	43	7·0638	13	13·0356	43	43·1177
14	14	2	17·9906	14	14	2·2998	44	44	7·2281	14	14·0383	44	44·1205
15	15	2	27·8471	15	15	2·4641	45	45	7·3924	15	15·0411	45	45·1232
16	16	2	37·7036	16	16	2·6284	46	46	7·5566	16	16·0438	46	46·1259
17	17	2	47·5600	17	17	2·7927	47	47	7·7209	17	17·0465	47	47·1287
18	18	2	57·4165	18	18	2·9569	48	48	7·8852	18	18·0493	48	48·1314
19	19	3	7·2730	19	19	3·1212	49	49	8·0495	19	19·0520	49	49·134
20	20	3	17·1295	20	20	3·2855	50	50	8·2137	20	20·0548	50	50·1369
21	21	3	26·9859	21	21	3·4498	51	51	8·3780	21	21·0575	51	51·1396
22	22	3	36·8424	22	22	3·6140	52	52	8·5423	22	22·0602	52	52·1424
23	23	3	46·6989	23	23	3·7783	53	53	8·7066	23	23·0630	53	53·1451
24	24	3	56·5554	24	24	3·9426	54	54	8·8708	24	24·0657	54	54·1479
				25	25	4·1069	55	55	9·0351	25	25·0685	55	55·1506
				26	26	4·2711	56	56	9·1994	26	26·0712	56	56·1533
				27	27	4·4354	57	57	9·3637	27	27·0739	57	57·1561
				28	28	4·5997	58	58	9·5279	28	28·0767	58	58·1588
				29	29	4·7640	59	59	9·6922	29	29·0794	59	59·1615
				30	30	4·9282	60	60	9·8565	30	30·0821	60	60·1643

112. How to find the logarithm of a small angle.

If θ be the circular measure of a small angle and n the number of seconds in it then—

$$\theta = \frac{n\,\pi}{180 \times 60 \times 60} = n \times \sin 1''.$$

$$\therefore \log\ \theta = \log n + \log\ \sin\ 1'' \text{ where } \log\ \sin\ 1'' = \log \frac{\pi}{180 \times 60 \times 60}$$

$$= \bar{6}{\cdot}6855749.$$

$$\text{Now} \sin\theta = \theta - \frac{\theta^3}{\angle 3} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$\therefore \frac{\sin\theta}{\theta} = 1 - \frac{\theta^2}{6} + \ldots\ldots\ldots = \left(1 - \frac{\theta^2}{2}\right)^{\frac{1}{3}} \text{approx.} = (\cos\theta)^{\frac{1}{3}} = (\sec\theta)^{-\frac{1}{3}}$$

$$\therefore \log\sin\theta = \log\theta - \tfrac{1}{3}\log\sec\theta = \log n + \log\sin 1'' - \tfrac{1}{3}\log\sec\theta.$$

Example.—Find the log of sin 39″.

$$\text{Log} \sin 39'' = 1{\cdot}5910646 + \bar{6}{\cdot}6855749 - \tfrac{1}{3}\,(0\ 0000000) = \bar{4}{\cdot}2766395.$$

$$\text{Similarly} \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\theta - \frac{\theta^3}{\angle 3} + \ldots\ldots}{1 - \frac{\theta^2}{\angle 2} + \ldots\ldots} = \theta\,\frac{1 - \frac{\theta^2}{6}}{1 - \frac{\theta^2}{\angle 2}}$$

$$\therefore \frac{\tan\theta}{\theta} = 1 + \tfrac{1}{3}\,\theta^2 + \ldots\ldots\ldots = \left(1 - \frac{\theta^2}{2}\right)^{-\frac{2}{3}} = \cos\theta^{-\frac{2}{3}} = \sec\theta^{\frac{2}{3}}$$

$$\therefore \log\tan\theta = \log\theta + \tfrac{2}{3}\log\sec\theta = \log n + \log\sin 1'' + \tfrac{2}{3}\log\sec\theta.$$

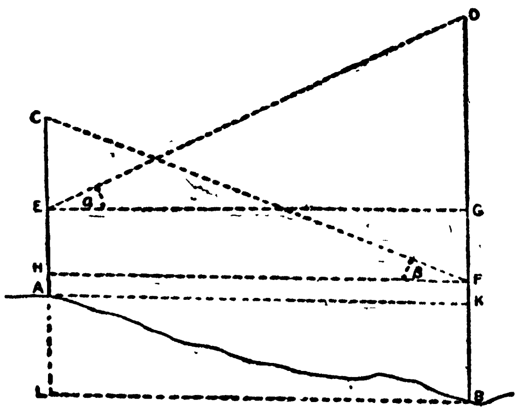
Example.—Find the log of tan 39″.

$$\text{Log} \tan 39'' = 1{\cdot}5910646 + \bar{6}{\cdot}6855749 + \tfrac{2}{3}\,(0\ 0000000) = \bar{4}{\cdot}2766395.$$

Values of small angles for degrees and minutes are found in ordinary log tables but the values of small angles to seconds must be computed as above since interpolation of such from log tables would be incorrect.

113. Another solution for reciprocally observed altitudes.

Fig. 34.



Given that reciprocal and simultaneous observations at two stations A and B have been made on both faces of the instrument, and that the

heights of signals C and D and also the heights of the instrument at telescope axes E and F above A and B have been recorded, and that EG, HF, AK and LB are all horizontal lines, and that EG = HF = AK = LB horizontal distance between A and B at level of A = d.

Let AC = height of signal C from the ground level = h_a
BD = „ „ „ D „ „ „ „ = h_b
AE = „ „ instrument „ „ „ at A = i_a
BF = „ „ „ „ „ „ B = i_b.

Let observed mean altitude of D at A = $\angle$ DEG = α.
„ „ „ „ C at B = $\angle$ CFB = β.

Then DG = $d \tan \alpha$.
CH = $d \tan \beta$.

BK = AL = diff. of level A and B.

But BK = BD − DG − GK = BD − DG − AE
$= h_b - d \tan \alpha - i_a$

and AL = CL − AC = CH + HL − AC
= CH + BF − AC
$= d \tan \beta + i_b - h_a$

Adding 2 BK = 2AL = $(d \tan \beta + i_b + h_b) - (d \tan \alpha + i_a + h_a)$.

Thus the difference of level between A and B

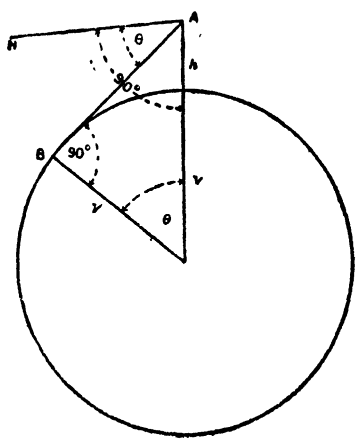
$$= \tfrac{1}{2} \{(d \tan \beta + i_b + h_b) - (d \tan \alpha + i_a + h_a)\}$$

It is noticed that the quantities which are named after a station, *e.g.*, $\tan \alpha$, i_a, h_a named after A, have the same sign.

Whether A or B is higher can easily be determined by the observer when he is at either station. Then the quantities belonging to the higher station are subtracted from the quantities belonging to the lower station: half the difference is the true difference of level between the two stations.

114. **Dip of the horizon.**—In figure let θ be the angle subtended at the centre of the earth, and since

Fig. 35.



$AB = \sqrt{(h+r)^2 - r^2} = \sqrt{h^2 + 2rh}$

and since also h^2 is very small compared with r then

$\therefore AB = \sqrt{2rh}$ very nearly.

Also $\tan \theta = \dfrac{AB}{r} = \sqrt{\dfrac{2rh}{r^2}} = \sqrt{\dfrac{2h}{r}}$

$$= \sqrt{\frac{2h}{1980 \times 5280}}$$

Now h is the height above M. S. L. of the observer in feet and r = radius of the earth in feet.

$\therefore \log \tan \theta = \tfrac{1}{2} \log 2h - \tfrac{1}{2} \log (1980 \times 5280)$
and if $h = 10$ feet then $\log \tan \theta = 7{\cdot}14087$

$\therefore \theta = 4' \, 45''$

115. **To determine the value of one division of the level scale on a theodolite.**—Roughly level the instrument and with the antagonistic screws bring one end of the bubble to a certain division, say 20, and with the vertical arc slow-motion screw intersect some object and record reading—let the reading be 0° 41′ 20.″ Again, with the antagonistic screws bring the same end of the bubble to division 0 so that the same end of the bubble has now travelled over 20 divisions. With the vertical arc slow-motion screw reintersect the object and let the reading be 0° 38′ 40.″ Then the difference of the readings in seconds divided by the number of the divisions travelled over by the bubble is equal to the value of one division of the bubble er level = $\frac{0° 41' 20'' - 0° 38' 40''}{20} = \frac{160''}{20} = 8''$.

To assign the correction for the bubble to vertical angle observations—

Suppose the following to be a record of a field-book and that the bubble division has a value of 20″ :—

A.	B.	*Level Reading.* Object end.	Eye end.
L 23° 26′ 00″	26′ 30″	7	5
R 24° 27′ 00″	28′ 00″	10	0
R 27° 05′ 00″	06′ 00″	7	4
L 25° 00′ 00″	00′ 00″	12	0

The mean angle is equal to the mean of the 8 readings plus or minus the correction for level. Now there are 4 level readings for object end totalling to 36 and 4 readings for eye end totalling to 9 and the correction is found as follows :—

$$\frac{\text{O} - \text{E}}{\text{number of readings}} \times \text{value of 1 division} = \frac{36 - 9}{8} \times 20 = \frac{27}{8} \times 20 = 1' \, 07.5''.$$

If the object end is in excess the correction must be added, and if the eye end is in excess the correction must be subtracted.

The mean angle is therefore 24° 59′ 19″ + 0° 1′ 07·5′

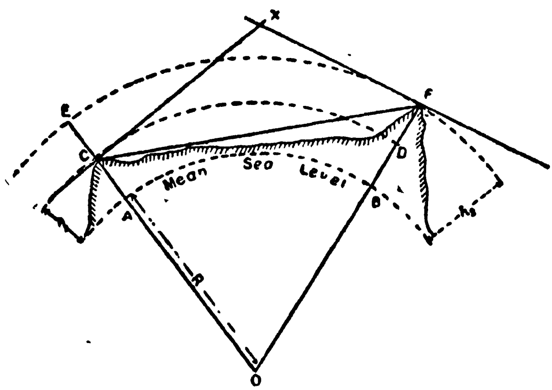
= 25° 00′ 26·5″.

The above is for a bubble graduated from the centre outwards ; if the bubble is graduated from one end only the readings of the zero end are considered negative and the above rule is then applied.

116. **Base line reduction to Mean Sea Level.**—Long lines such as base lines and sides of triangulation must be reduced to M.S.L. to obtain the Geodetic distance.

Let a base line CD be measured and required to be reduced to horizontal distance. Let the heights of C and F be 2,640 and 5,280 feet respectively above mean sea level and be represented by h_1 and h_2.

Fig. 36.



Then $\frac{AB}{CD} = \frac{R}{R + h_1}$

and $\frac{AB}{EF} = \frac{R}{R + h_2}$ where R is the mean radius of the earth.

The mean of the polar and equatorial diameters is usually taken as 7912·03 miles, and therefore the mean radius = 3956·015 or R = 20,899,318 feet; its log being 7·3201319.

$$\text{Then } \frac{AB}{5 \text{ miles}} = \frac{3956}{3956{\cdot}5}$$

$$\therefore AB = \frac{5\,(3956)}{3956{\cdot}5}$$

$$\log AB = 4{\cdot}2962263 - 3{\cdot}5973112$$
$$= 0{\cdot}6989151$$
$$\therefore AB = 4{\cdot}99937 \text{ miles}$$

or ·00063 miles less than 5 miles

= 16·63′ less than 5 miles measured on a plane half of a mile above Mean Sea Level.

117. **Probable errors**.—The arithmetic mean of a number of observations is the most *probable value* of the quantity observed, and the *probable error* in such a quantity is that it is an even wager that it is so much more or so much less of the real value.

The deduction of probable errors is useful as it shows whether the errors are within the allowable error, and how much or to what extent a certain set of observations may be depended on.

It must be remembered that constant errors which can be eliminated must be first eliminated before the method of least squares is applied. A tape that is known to be too long will give short records of measurements and these measurements must first be corrected before any "weight" can be assigned to the accuracy of any set of measures. Any observation which is obviously wrong must be eliminated, and one of the assumptions is that large errors do not occur, but what is

frequent is that a plus and minus error of reading, etc., cancel. In fact, to summarise and attach any weight to sets of observations the conditions of each set must more or less be equal.

If n = number of observations.

d = the difference between any one observation and the arithmetic mean.

E = the probable error of any one observation.

E_0 = the probable error of the mean of all the observations.

c = 0·6745 a constant found by the theory of least squares.

Σ = "the sum of."

Then from the theory of least squares.

$$E = 0{\cdot}6745\sqrt{\frac{\Sigma d^2}{n-1}}$$

$$E_0 = 0{\cdot}6745\sqrt{\frac{\Sigma d^2}{n(n-1)}}$$

Example.—Abstract of angles taken from a triangulation field-book.

Angle.				*d.*	*d²*.
°	′	″			″
70	55	12	+	7	49
„	54	59	−	6	36
„	55	06	+	1	1
„	55	03	−	2	4
Mean 70°	55′	05″			90

$$\text{Then } E = 0{\cdot}6745\sqrt{\frac{90}{4-1}} = 0{\cdot}6645\sqrt{30}$$
$$= 3{\cdot}7''.$$

$$E_0 = 0{\cdot}6475\sqrt{\frac{90}{4(4-1)}} = 0{\cdot}6745\sqrt{\frac{90}{12}}$$
$$= 1{\cdot}85''.$$

$$\text{angle} = 70°\ 55'\ 05'' \pm 1{\cdot}85''.$$

Probable errors give an idea as to what *weight* should be given to different sets of observations, and these *weights*, it has been found, vary inversely as the squares of the probable errors.

If, for example, another set of observations had been taken and E_0 was found to be 2·57″ the *weights* of the two observations would be as $\frac{1}{(1{\cdot}85)^2} : \frac{1}{(2{\cdot}57)^2}$ or as 2 : 1.

In *traversing* from azimuth to azimuth the probable error in any angle will be equal to the total error divided by the *square root* of the number of angles of the traverse.

Example.—If in 36 stations the angular error in closing was 6 minutes the error per angle is not as is generally supposed $\frac{6}{36}'$ or 10 seconds but $\sqrt{\frac{6'}{36}} = 1$ minute.

The reason for this is that errors are compensating and thus a fictitious closing error is arrived at and not a true one showing the accumulation of error.

In *measuring* if the following were the records of measures after being reduced for constant errors the probable error per whole measurement will be found as follows :—

	d.	*d²*.
516·7	·425	·180625
516 3	·025	·000625
515·9	·375	·140525
516·2	·075	·005525
Mean = 516·275		Σd^2 = ·327300

$$E_o = 0{\cdot}6745\sqrt{\frac{{\cdot}327300}{4 \times 3}} = 0{\cdot}6745\sqrt{\frac{{\cdot}327300}{12}}$$

$$= 0{\cdot}6745\sqrt{{\cdot}025608} = {\cdot}108$$

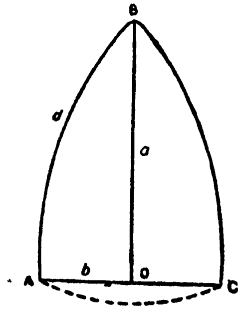
or the probable error of the mean is about $\frac{1}{4780}$.

118. **To lay down a parallel of latitude.**—If an azimuth is observed on a certain latitude and a line laid down with a direction 90° from the true meridian, then this line will at the place, make a great circle through the point ; and a parallel of latitude, which is a small circle, will have a direction less than 90° or 90° minus the convergency correction for the distance of the next point on that latitude.

To illustrate this, imagine a parallel of latitude and at numerous points along it azimuths are taken. All these azimuths will converge and meet in a point on the earth's axis above the pole, and the shape that these lines will take will be that of a cone with the parallel of latitude as its base. At the equator these lines would take the shape of a cylinder intersecting the earth at the equator. Therefore the convergency angle for the latitude and for a certain distance along the latitude must be found and subtracted from 90° to obtain the initial direction of the parallel of latitude. The measure or distance taken is then pegged out and another azimuth is taken, and so on.

A simpler way is as follows :—

Fig. 37.



Take the spherical triangle ABC and let A and C, for example, be points on latitude 30° N. and 1° apart in longitude (1° apart in longitude is equal to 60 nautical miles) and a nautical mile is that portion of the arc of a great circle subtended by one minute at the centre of the earth on the surface of the earth at mean sea level. At the equator it is equal to 6085·8 feet.*

From B let drop a perpendicular to meet AC in D. Then the triangle ABD is right-angled at D and according to Napier's rule of circular parts :—

$$\begin{aligned}
\operatorname{Sin}\left(\tfrac{\pi}{2} - A\right) &= \tan\left(\tfrac{\pi}{2} - d\right) \tan b \ (\textit{see}\ \text{para. } 61\ldots\ldots) \\
&= \cot d \tan b \\
\text{Log} \cos A &= \log \cot 60^\circ + \log \cot \tfrac{1}{2}^\circ \\
&= \bar{1}{\cdot}7614394 \\
&+ \bar{3}{\cdot}9408584 \\
\text{Log} \cos A &= \bar{3}{\cdot}7022978 \\
\therefore A &= 89^\circ\ 42'\ 39''
\end{aligned}$$

and $\therefore C = 89^\circ\ 42'\ 39''$,

that is, a line AC laid down making an angle 89° 42′ 39″ with the true N. at A will pass through C.

Instead of finding the angle A or C the convergency can be found, *vide* para. 131, Part I., Survey Manual, as follows :—

Log convergency in minutes = log constant for feet + log tan latitude + log departure = $\bar{4}{\cdot}2164$ log constant.

9·7614 log tan 30°.

5·2607 log departure.

log convergency = 1·2385

$\therefore$ convergency = 17′ 19″ or angle = 90° − 0° 17′ 19″ = 89° 42′ 41″ the difference of a second or so is due to the logs being taken to four places only, but the latter value is quite near enough as no theodolite used by the ordinary surveyor will read to accuracy in seconds.

Now to compute the point D we take a as the co-latitude of the middle point D and thus $\cos a \cos b = \sin\left(\frac{\pi}{2} - d\right) \therefore \cos a = \frac{\cos d}{\cos b}$

$$\therefore a = 59^\circ\ 59'\ 55''.$$

* *Vide* Auxiliary Tables, Survey of India, 4th Ed.

$\therefore$ latitude of point D = 30° 0′ 5″, therefore the point D is 5″ N. of the parallel of latitude 30°.

Now, one minute of arc subtends 6085·8 feet as shown above

$\therefore$ 5 seconds subtend $\frac{6086}{12}$ feet.

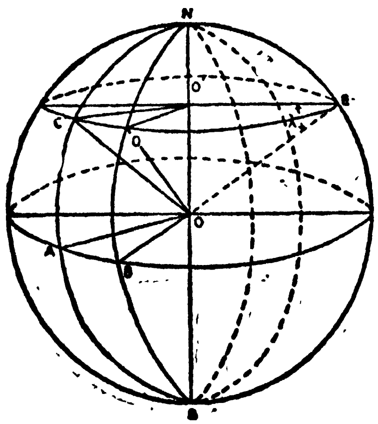
= 507 feet nearly, or 507 feet will be the offset south at a point 30 nautical miles distant from either A or C = 30 × 5275·11 feet (*see* next problem) = 158,253 feet along the parallel of latitude. The arc ADC is that of a great circle or the shortest line between A and C through D ; and in the triangle ABD, sin AD = sin B sin *d* $\therefore$ AE = 25·98 minutes $\therefore$ AD in feet = 6085·8 × 25·98 = 158,109. The parallel of latitude thus becomes the hypotenuse, the part of the great circle the base and the ordinate or offset 507 feet the perpendicular of a triangle. One or more offsets may be computed in the same manner and the remainder put in by interpolation of values for similar triangles.

This method should be remembered by those called upon to lay down townships, canals, colonies, etc.

In the U.S.A. the parallels are usually projected by the secant method and tables have been made to facilitate such work.

119. **The Nautical mile.**—A nautical mile subtends, at the equator, an angle of 1 minute at the centre of the earth or more correctly is a minute in arc of a great circle.

Fig. 38.



If the mean radius* of the earth is 3956·46 miles we obtain the length of a nautical mile in feet to be $\frac{\pi \times 3956{\cdot}46 \times 5280}{180 \times 60}$ = 6,079 feet very nearly at mean sea level at the equator.

If EDC (*see* Fig. 36) is a parallel of latitude and EO = R is the radius of the earth and EO′ = r = the radius of the circle EDC then r = R cos O′ EO = R cos latitude or $\cos\lambda = \frac{r}{R}$.

Now if NCA and NDB are two great circles, then the difference in longitude (L) of D and C in latitude (λ) is measured by the arc AB on the equator and $\frac{\text{arc DC}}{r} = \frac{\text{arc AB}}{R}$ = circular measure of L. or arc DC = $\frac{r \times \text{arc AB}}{R}$.

* The value of the mean radius is here accepted but in laying out a parallel of latitude on the surface of the earth the equatorial radius only must be accepted. The following values are given :—Mean semi-diameter at equator = 20925867 feet and mean semi-polar diameter = 20854477 feet $\therefore$ mean radius = 20890172 feet.

and if x = the number of minutes in L then the distance AB $= x$ nautical miles,

or $DC = \frac{rx}{R}$ nautical miles.

$= x \cos \lambda$ nautical miles.

Therefore a nautical mile in latitude $30° = 5264{\cdot}7$ feet, and thus when calculating the lower and upper parallels for the graticules of maps it will be found that the upper parallel will be shorter than the lower parallel for all maps projected for a graticule north of the equator and *vice versâ* for south of the equator.

Again, D and C are points situated on the same latitude (λ) and let their difference in longitude (L) be 40° or 2,400 minutes. If a ship is travelling at 17 knots (17 nautical miles an hour) due west or east between D and C, and if D and C were in a latitude of 50° she would do the journey in $\frac{2400 \times \cos 50°}{17}$ hours

$= 90{\cdot}7$ hours.

The chord $DC = 2\,r \sin \frac{40°}{2} = 2\,r \sin 20°$.

and chord $DC = 2\,R \sin \frac{\theta}{2}$ (where θ = angle COD)

and since $r = R \cos \lambda$

$\therefore \sin\frac{\theta}{2} = \sin 20° \cos 50°$

$\therefore \frac{\theta}{2} = 12° \, 42'$

$\therefore \theta = 1{,}524$ minutes,

and therefore the arc CD of the great circle = 1,524 nautical miles, and thus if the course of the ship had been on the great circle instead of due east and west she would have done the journey in $\frac{1524}{17}$ hours or 89·7 hours or one hour less.

120. **Micrometers.**—The superior class of theodolites or what may be termed precision theodolites are fittted with micrometers instead of verniers. The primary limb of the theodolite is divided into degrees and $\frac{1}{6}$th of degrees or 10 minutes. The micrometer attachment consists of a box placed about midway between its lenses. In this box is fitted a "comb" as it is called with a **V** notch which is the index or reading. On the right hand side of the box there is a wheel which being revolved moves a pair of parallel wires across the field of view. One revolution of the wheel should move the parallel wires from one graduation on the primary scale to the next. If the value of one graduation to the next is 10 minutes then it can be seen that if the wheel is divided into

10th minutes are obtained, and if again into 6ths that 10 secs. are obtained and an approximation to one second is possible.

The following will simplify matters as regards how to read the micrometer :—

First focus the eye-piece so that the graduations and wires are clear and distinct and bring the wires to the centre of the V notch when the micrometer wheel if properly set should read 0 ; if it does not, with the right hand pull out the spring button of the wheel and turn the wheel so that the 0 graduation is opposite its own index and release the button. A few trials will settle this. Open the clamp screw of the upper plate of the theodolite and with the slow-motion screw bring 0° directly in the centre of the notch. The theodolite is now reading 0° 0′ 0″. Now turn the slow-motion screw to read something more than 0°, and say less than 0° 10 minutes, that is, the notch will be moved to the right of 0°. Turn the micrometer wheel by which the wires are moved and let the wires be placed one on each side of the graduation for 0° or 0° 0′ whichever is the closer. Examine the micrometer wheel and if the index is opposite some division between 6 and 7 on the wheel the reading will be something between 0° 6′ and 0° 7′. To continue yet further if the index pointed midway between the 2nd and 3rd division beyond 6 on the micrometer wheel the reading would be 0° 6′ 25″. Take another case—the notch is between the 4th and 5th division to the right of 265° and the micrometer wheel points to 7′ 36″ ; the reading is therefore 265° 47′ 36″.

What is known as the "run" of the micrometer is whether the wires travel from one graduation to another in exactly one revolution of the wheel, and this depends on the focus of the lower lens and it will be necessary to increase or decrease the distance by means of the screw and collar for that purpose till by repeated trials the run is adjusted. This being completed the focus on looking through the eye-piece will have been altered when it is necessary to adjust by sliding the whole attachment in its socket up and down.

So that the two micrometers, one on each side, should differ by 180° the comb can be moved laterally by a screw fitted for the purpose on one side of the box.

The above are what might be termed permanent adjustments and are not often necessary, but the adjustment given earlier that of making the 0 of the micrometer agree with the V notch comb is easily made and is sometimes necessary ; the permanent adjustment of the comb for instance will necessitate the adjustment for the zero of micrometer wheel.

The micrometer is much easier to read than the vernier and is less strain on the eye-sight, and but for its cost, its adoption would be universal.

INDEX TO PARAGRAPHS.